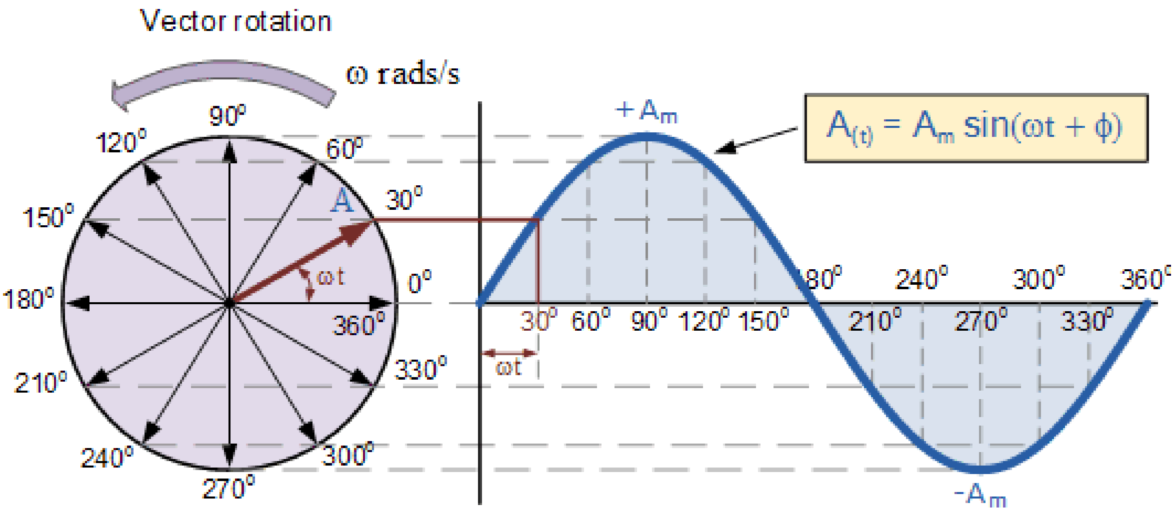
Module-3

Phasor representation of an Alternating Quantity

As we know that the alternating quantities of voltage or current are vector quantities having both magnitude and direction. But in case of instantaneous values are continuously changing so it can be represented by a rotating vector or phasor.so we can define a phasor is a vector rotating at constant angular velocity.



Rotating Phasor

Sinusoidal Waveform in the Time Domain

Here at time t1, $\omega t_1 = 30^0$, $v_1 = v_m \sin \omega t_1 = OA \sin \omega t_1 = OA \sin 30^0$

$t_2, \omega t_2 = 60^0$, $v_1 = v_m \sin \omega t_2 = OA \sin \omega t_2 = OA \sin 60^0$

$t_3, \omega t_3 = 90^0$, $v_1 = v_m \sin \omega t_3 = OA \sin \omega t_3 = OA \sin 90^0$

$t_4, \omega t_4 = 120^0$, $v_1 = v_m \sin \omega t_4 = OA \sin \omega t_4 = OA \sin 120^0$

$t_5, \omega t_5 = 150^0$, $v_1 = v_m \sin \omega t_5 = OA \sin \omega t_5 = OA \sin 150^0$

$t_6 . \omega t_6 = 180^0$, $v_1 = v_m \sin \omega t_6 = OA \sin \omega t_6 = OA \sin 180^0$

And so on.

Consequently ,the phasor having magnitude v_m and rotating in anticlockwise direction at an angular velocity ωt represents the sinusoidal voltage $v = v_m \sin \omega t$

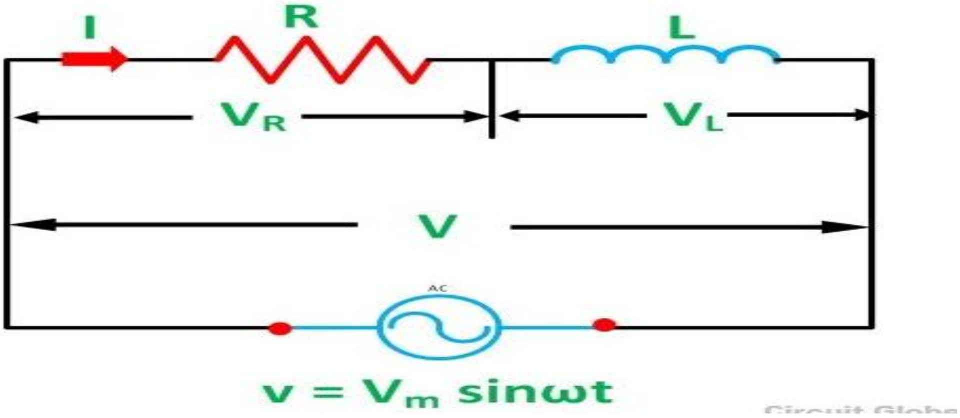
Phasor diagram:

The phasor diagram is one in which different alternating quantities of the same frequency are represented by phasor with their correct relationship.

The phasor representing two or more alternating quantities of the same frequency rotate in counter-clock wise direction with the same angular velocity, thereby maintaining a fixed position with respect to one another.

A.C Through Pure Resistance and Inductance in series Circuit

Let V and I be the rms value of voltage and current respectively in the given circuit.



$$\overline{V} = \overline{V_R} + \overline{V_L}$$

$$V = \sqrt{V_R^{\ 2} + V_L^{\ 2}}$$

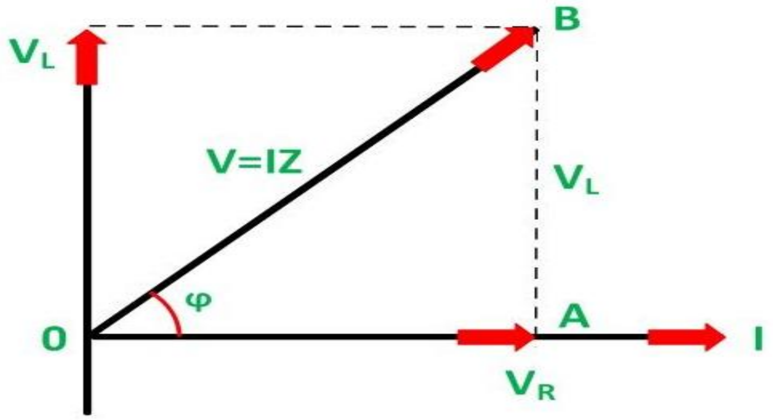
$$V = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + X_L^{\ 2}}$$

$$I = \frac{V}{\sqrt{R^2 + X_L^{\ 2}}} = \frac{V}{Z_{RL}}$$

Where $Z_{RL} = \sqrt{R^2 + X_L^{\ 2}}$ =Impedance in R-L circuit

The phasor diagram of R-L circuit

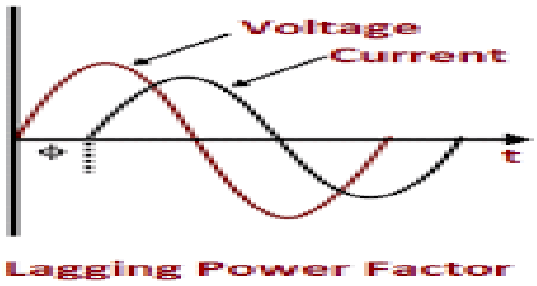


$$\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{\omega L}{R}$$

$$\phi = \tan^{-1}(\frac{\omega L}{R})$$

From the phasor diagram, Current I lag behind the supplied voltage V by an angle ϕ

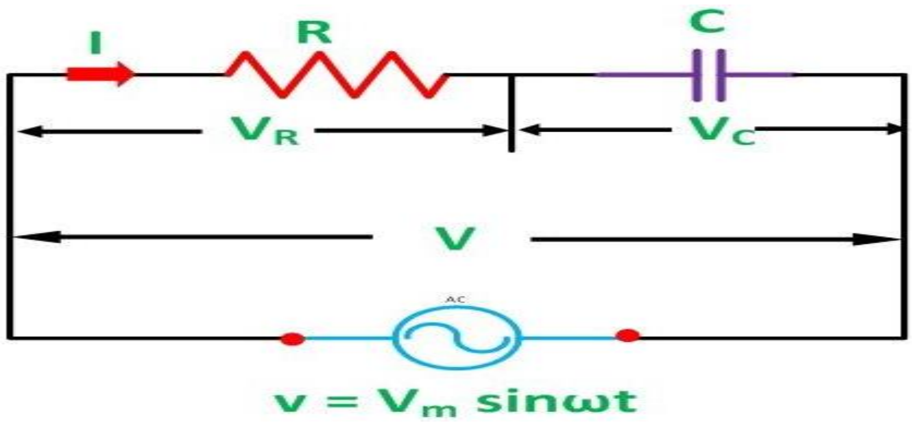
If voltage $V = V_m \sin\omega t$ then the resulting current in the circuit $I = I_m \sin(\omega t - \phi)$



Power factor: $\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ_{RL}} = \frac{R}{Z_{RL}}$ =power factor of this circuit

A.C Through Pure Resistance and Capacitance in series Circuit

Let V and I be the rms value of voltage and current respectively in the given circuit.



$$\overline{V} = \overline{V_R} + \overline{V_C}$$

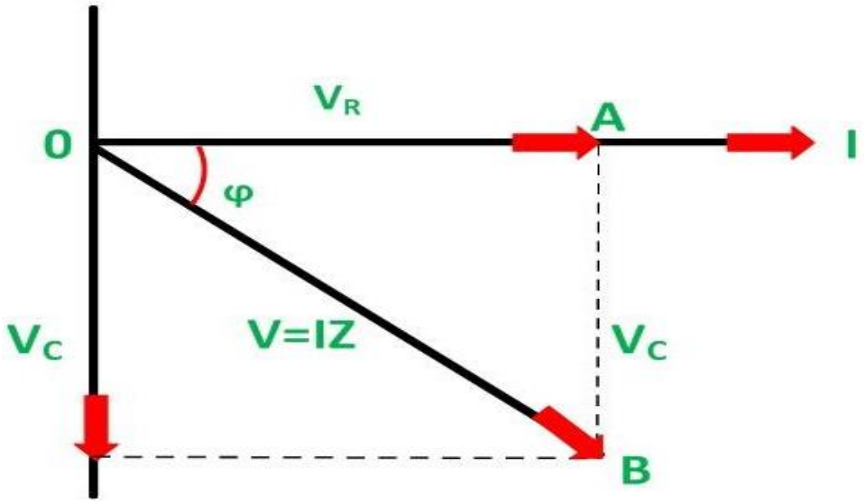
$$V = \sqrt{V_R^{\;2} + V_C^{\;2}}$$

$$V = \sqrt{(IR)^2 + (-IX_C)^2}$$

$$V = I\sqrt{R^2 + X_C^{\;2}}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^{\;2}}} = \frac{V}{Z_{RC}}$$

Where $Z_{RC} = \sqrt{R^2 + X_C{}^2}$ =Impedance in R-C circuit



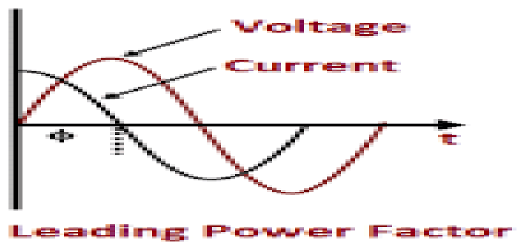
The phasor diagram of R-C circuit

$$\tan\phi = \frac{V_C}{V_R} = \frac{-IX_C}{IR} = \frac{-X_C}{R} = \frac{-1}{\omega RC}$$

$\phi = \tan^{-1}(\frac{-1}{\omega RC})$ where $X_C = \frac{1}{\omega C}$

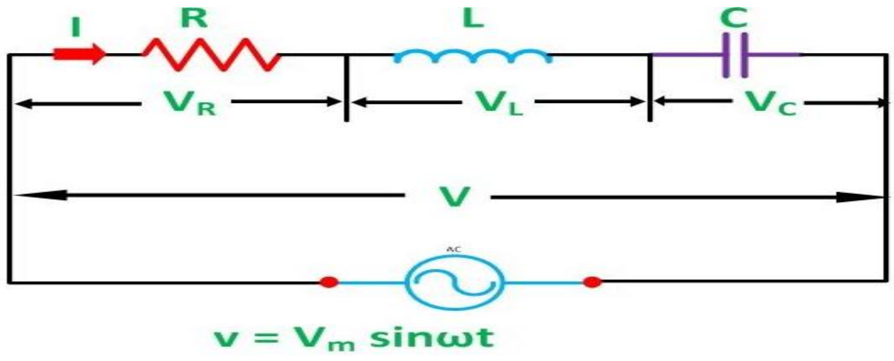
If voltage $V = V_m \sin \omega t$ then the resulting current in the circuit $I = I_m \sin(\omega t + \phi)$

Power factor: $\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ_{RC}} = \frac{R}{Z_{RC}}$ =power factor of this circuit



A.C Through Pure Resistance, Inductance and Capacitance in series Circuit

Let V and I be the rms value of voltage and current respectively to the R-L-C series circuit.



Then

$$\overline{V} = \overline{V_R} + \overline{V_L} + \overline{V_C}$$

$$V = \sqrt{V_R^{\,2} + V_L^{\,2} + V_C^{\,2}}$$

$$V = \sqrt{(IR)^2 + (IX_L)^2 + (IX_C)^2}$$

$$V = I\sqrt{R^2 + X_L^{\,2} + X_C^{\,2}}$$

$$I = \frac{V}{\sqrt{R^2 + X_L^{\,2} + X_C^{\,2}}} = \frac{V}{Z_{RLC}}$$

Where $Z_{RLC} = \sqrt{R^2 + X_L^{\,2} + X_C^{\,2}}$ =Impedance in R-L-C circuit

Phasor diagram:

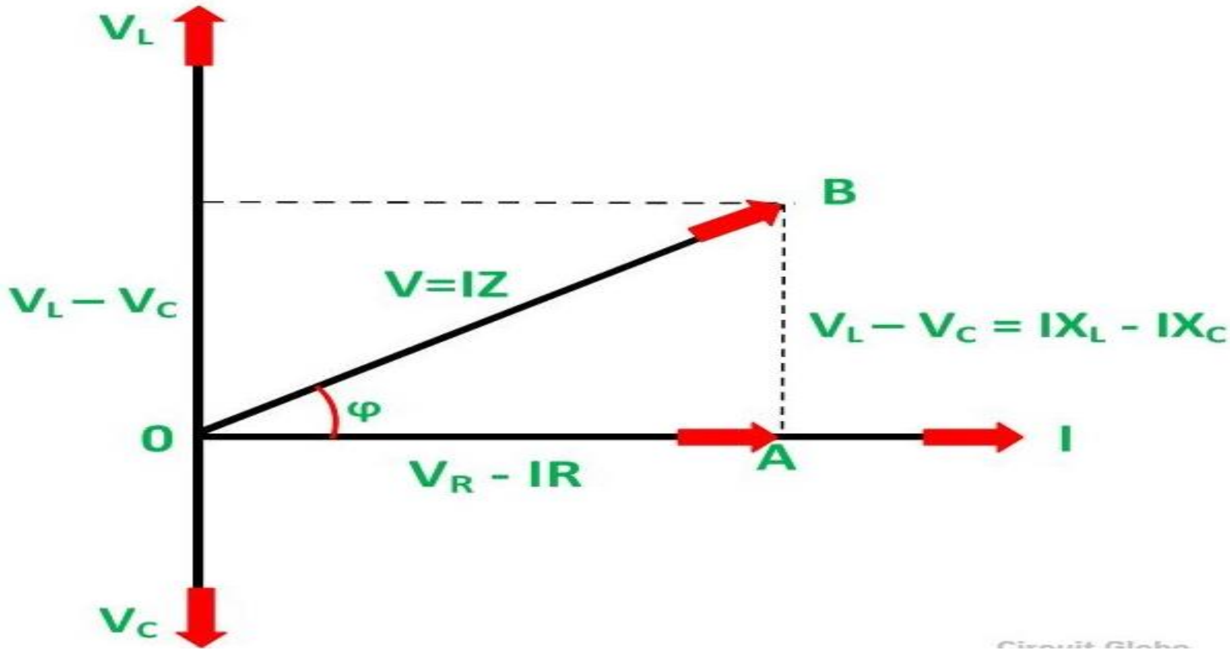
As V_L and V_C are two vector 180^0 out of phase with each other ie opposite direction of vectors.

Here the phasor diagram can be obtained in two different cases

Case-1: if $X_L > X_C$, $IX_L > IX_C$ that means $V_L > V_C$ the net voltage is $V_L - V_C$

Case -2: if $X_L < X_C$, $IX_L < IX_C$ that means $V_L < V_C$ the net voltage is $V_C - V_L$

Case-1 phasor diagram as shown in the Figure



$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{Z_{RLC}}$$

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R}$$

$$\phi = \tan^{-1}[\frac{X_L - X_C}{R}]$$

If $V = V_m \sin \omega t$ then the resulting current in the circuit $I = I_m \sin(\omega t - \phi)$

Similarly, in the case-2

$$I = \frac{V}{\sqrt{R^2 + (X_C - X_L)^2}} = \frac{V}{Z_{RLC}}$$

$$\tan\phi = \frac{V_C - V_L}{V_R} = \frac{IX_C - IX_L}{IR} = \frac{X_C - X_L}{R}$$

$$\phi = \tan^{-1}[\frac{X_C - X_L}{R}]$$

If $V = V_m \sin \omega t$ then the resulting current in the circuit $I = I_m \sin(\omega t + \phi)$

$$\cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$$ =power factor of the circuit

Admittance of A.C circuit:

Admittance Y is the reciprocal of impedance Z of the A.C circuit.

$$Y = \frac{1}{Z} = \frac{1}{V/I} = \frac{I}{V}$$

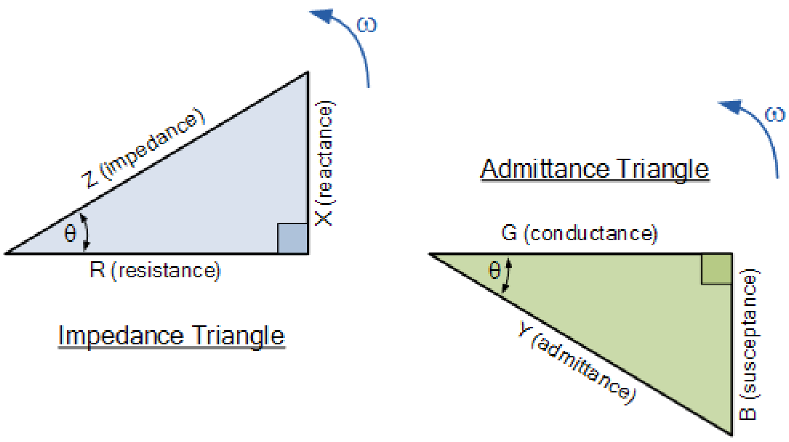
$Y = \frac{1}{Z} = G \pm jB$ where G=real part of the admittance =Conductance

B=Imaginary part of the admittance = Susceptance

$G = Y\cos\theta$ =conductance $= \frac{1}{Z}\frac{R}{Z} = \frac{R}{Z^2}$

$B = Y\sin\theta$ =susceptance $= \frac{1}{Z}\frac{X}{Z} = \frac{X}{Z^2}$

$$Y = \sqrt{G^2 + B^2}$$



RMS Value:

The average value of this square wave I $= \frac{i_1^2 + i_2^2 + i_3^2 + \dots\dots\dots + i_n^2}{n}$

The root of this expression is the root mean square (RMS)

$$I_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + \dots\dots\dots + i_n^2}{n}}$$

The General expression of the rms value of the any periodic function f(t) with a period T is

$$G_{rms} = \sqrt{\frac{1}{T}\int_0^T [f(t)]^2\,dt}$$

Ex. The rms value of sinusoidal current $I_{rms} = I_m \sin \omega t$ can be obtained as

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T [I_m \sin \omega t]^2 dt}$$

$$I_{rms} = \sqrt{\frac{I_m{}^2}{2T}\int_0^T [1-\cos 2\omega t] dt}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

Similarly, for the non-sinusoidal wave of current or any function can be obtained in rms value as follows

$$I = I_0 + I_{m1}\sin \omega t + I_{m2}\sin 2\omega t + I_{m3}\sin 3\omega t + ... + I_{mn}\sin n\omega t$$

$$I_{rms} = \sqrt{I_0{}^2 + I_{m1}{}^2 + I_{m2}{}^2 + ... + I_{mn}{}^2}$$

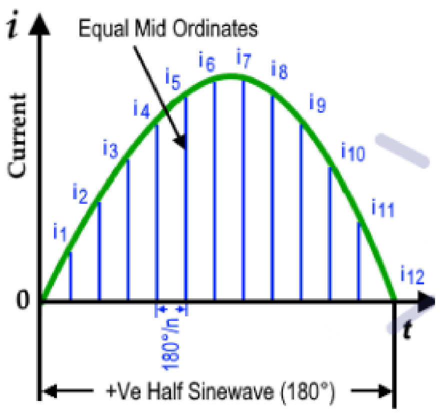
$$I_{rms} = \sqrt{I_0{}^2 + I_{1rms}{}^2 + I_{2rms}{}^2 + ... + I_{nrms}{}^2}$$

$$I_{rms} = \sqrt{I_0{}^2 + \frac{I_{m1}{}^2}{2} + \frac{I_{m2}{}^2}{2} + ... + \frac{I_{mn}{}^2}{2}}$$

Average Value:

The average value of an A.C current or voltage is the average of all the instantaneous values during one alteration. They are actually DC value.

$$I_{avg} = \frac{i_1 + i_2 + i_3 + ... + i_n}{n}$$



$$I_{avg} = \frac{i_1 + i_2 + i_3 + ... + i_{12}}{12}$$

The general expression for average value of any function f(t) having a periodic function with period T

$$G_{avg} = \frac{1}{T}\int_0^T f(t)dt$$

EX. The average value of sinusoidal current $I_{rms} = I_m \sin \omega t$ can be obtained as

$$I_{avg} = \frac{1}{T}\int_0^T I_m \sin \omega t dt = \frac{2I_m}{\Pi} = 0.637 I_m$$

$$Form\ Factor(FF) = \frac{rms value}{average value}$$

Average Power

Let the instantaneous voltage and current can be taken as

$$v(t) = v_m \cos(\omega t + \theta)$$

$i(t) = i_m \cos(\omega t + \phi)$ then the instantaneous power $p(t) = v(t) * i(t)$

$$p(t) = v_m \cos(\omega t + \theta) * I_m \cos(\omega t + \phi)$$

$$p(t) = v_m i_m [\frac{1}{2}\cos(\theta - \phi) + \frac{1}{2}\cos(2\omega t + \theta + \phi)]$$

The average power $P_{avg} = \frac{1}{T}\int_0^T p(t)dt$

$$P_{avg} = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T v_m i_m [\frac{1}{2}\cos(\theta - \phi) + \frac{1}{2}\cos(2\omega t + \theta + \phi)dt \qquad (1)$$

$$P_{avg} = \frac{1}{T}\int_0^T p(t)dt = \frac{v_m i_m}{2}\cos(\theta - \phi) = v_{rms} i_{rms} \cos(\theta - \phi)$$

$$P_{avg} = v_{rms} i_{rms} \cos(\theta - \phi)$$

In the equation (1) having two terms, 1st term is the independent of time function and 2nd term is sinusoid having frequency twice the frequency of original voltage and current waveform. The average power for the 2nd term for a full cycle is zero.

Complex Power:

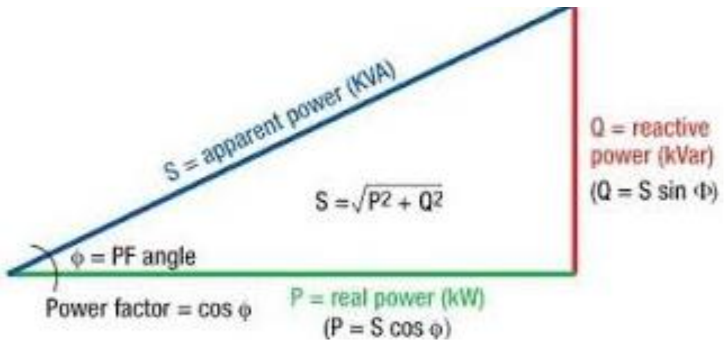
If the current and voltage of a circuit are given in phasor form then the complex power is

$$S = VI^* = P \pm jQ$$

Where $S = P + jQ$ for net inductive circuit

$S = P - jQ$ for net capacitive circuit

S is the total power or apparent power, P is active power and Q is reactive power

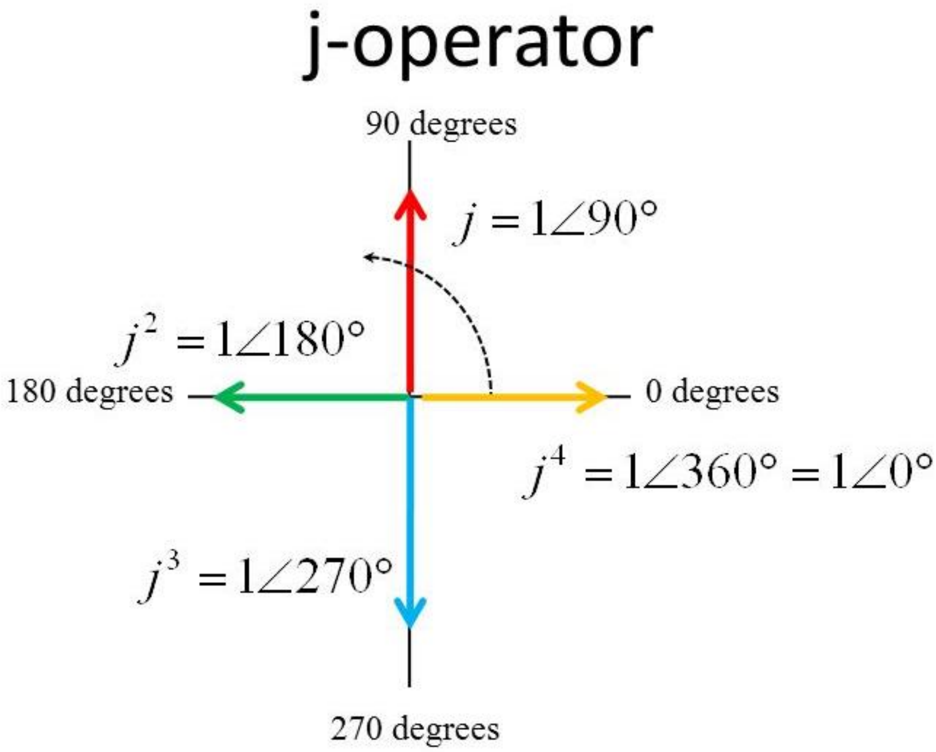


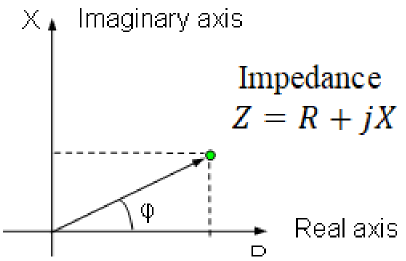
$\cos\phi = \frac{P}{S} = \frac{kW}{KVA}$ =power factor of the circuit

$P = S\cos\phi = V_{rms} I_{rms} \cos\phi$

$Q = S\sin\phi = V_{rms} I_{rms} \sin\phi$

Complex Notation of A.C circuit:





A vector can be written in different form

(a) Cartesian or rectangular form
(b) Trigonometrical form
(c) Polar form
(d) Exponential form

Let us consider a vector in **cartesian form** $Z = R + jX$

$|Z| = \sqrt{R^2 + X^2}$ = magnitude of the vector, $\varphi = \tan^{-1}[\frac{X}{R}]$ =phase angle, this phase angle can be measured from the positive real axis to the vector and taken as positive in counter clockwise direction and taken in negative in clockwise direction.

In this vector j is a operator which operate 90^0 , In counter clock wise direction taken as positive value.

Ex In cartesian vector can be addition or subtraction of two or more vector as follows

If $Z_1 = R_1 + jX_1 \quad Z_2 = R_2 + jX_2$

$$Z_1 + Z_2 = R_1 + jX_1 + R_2 + jX_2 = (R_1 + R_2) + j(X_1 + X_2)$$

$$Z_1 - Z_2 = R_1 + jX_1 - (R_2 + jX_2) = (R_1 - R_2) + j(X_1 - X_2)$$

The same vector can be written in **Trigonometrical form** as

$$\overline{Z} = |Z|\cos\varphi + j|Z|\sin\varphi$$

$$\overline{Z} = |Z|(\cos\varphi + j\sin\varphi)$$

Then this vector can write in **polar form** as

$$\overline{Z} = |Z|\angle\varphi$$

In polar form of vectors can be multiplied or divided by two or more vector as follows

$$\overline{Z_1} = |Z_1|\angle\varphi_1 \quad \overline{Z_2} = |Z_2|\angle\varphi_2$$

Multiplication of two polar vector $\overline{Z} = (|Z_1|\angle\varphi_1)(|Z_2|\angle\varphi_2) = |Z_1||Z_2|\angle\varphi_1 + \varphi_2$

Division of two polar vector $\frac{\overline{Z_1}}{\overline{Z_2}} = \frac{|Z_1|\angle\varphi_1}{|Z_2|\angle\varphi_2} = \frac{|Z_1|}{Z_2}\angle\varphi_1 - \varphi_2$

This vector also written in **exponential form** as

$\overline{Z} = |Z|e^{j\varphi}$

Representation of voltage, current and impedance in complex Notation

Let the voltage and current in an A.C circuit can be expressed in the form

$\overline{V} = |V|\angle\alpha$ and $\overline{I} = |I|\angle\beta$ where α and β are the angular displacement of $\overline{V}$ and $\overline{I}$ respectively from a reference direction.

(a) A.C Through pure Resistance

Let voltage phasor $\overline{V}$ be taken as reference phasor so $\overline{V} = |V|\angle 0^0$

But in case of pure resistive circuit, the current and voltage are in phase, so current taken as $\overline{I} = |I|\angle 0^0$

$$Z_R = \frac{\overline{V}}{\overline{I}} = \frac{|V|\angle 0^0}{|I|\angle 0^0} = \frac{V}{I}\angle 0^o = R\angle 0^0 = R + j0 = R$$

(b) A.C Through pure Inductance

In pure inductance, the current lags the voltage by 90^0

$\overline{I} = |I|\angle -90^0$

$$Z_L = \frac{\overline{V}}{\overline{I}} = \frac{|V|\angle 0^0}{|I|\angle -90^0} = \frac{V}{I}\angle 90^o = X_L\angle 90^0 = 0 + jX_L = j\omega L$$

(c) A.C Through pure capacitance

In pure capacitance, current leads the voltage by 90^0

$\overline{I} = |I|\angle 90^0$

$$Z_C = \frac{\overline{V}}{\overline{I}} = \frac{|V|\angle 0^0}{|I|\angle 90^0} = \frac{V}{I}\angle -90^o = X_C\angle -90^0 = 0 - jX_C = \frac{-j}{\omega C}$$

(d) A.C Through R-L circuit

The total impedance of a series RL A.C circuit is given by

$$Z_{RL} = Z_R + Z_L = (R + j0) + (0 + jX_L) = R + jX_L$$

(e) A.C Through R-C circuit

The total impedance of a series R-C A.C circuit is given by

$$Z_{RC} = Z_R + Z_C = (R + j0) + (0 - jX_C) = R - jX_C$$

(f) A.C Through R-L-C circuit

The total impedance of a series R-L-C A.C circuit is given by

$$Z_{RLC} = Z_R + Z_L + Z_C = (R + j0) + (0 + jX_L) + (0 - jX_C) = R + j(X_L - X_C)$$
$$Z_{RLC} = R + j(X_L - X_C) = R + jX$$

If two or more impedance are connected in series than total impedance of a series circuit is the phasor sum of the impedances of the circuit

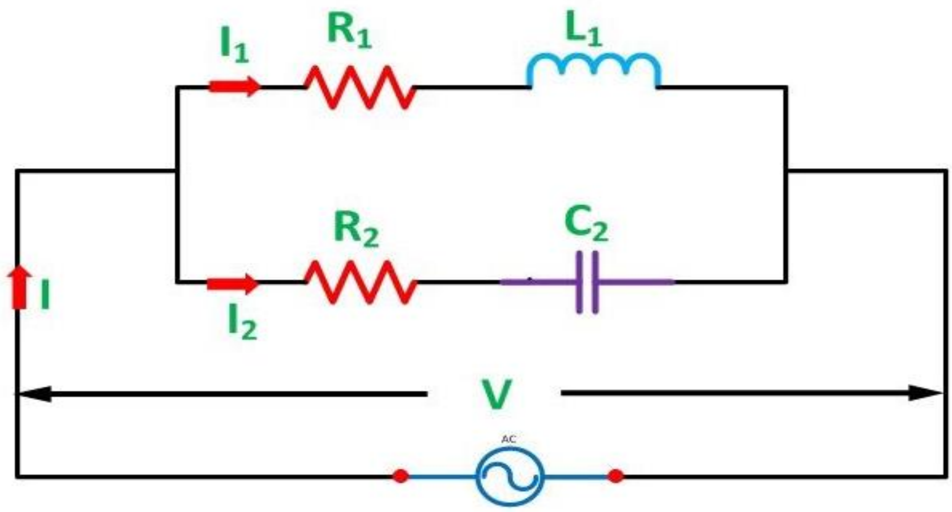
$$Z_T = Z_1 + Z_2 + Z_3 + \ldots\ldots\ldots\ldots + Z_n$$

If two or more impedance are connected in parallel than the total impedance can be obtained as

$$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots\ldots\ldots + \frac{1}{Z_n}$$
$$Y_T = Y_1 + Y_2 + Y_3 + \ldots\ldots\ldots\ldots + Y_n$$

Where Y_1, Y_2 and Y_3 are the admittances corresponding to Z_1, Z_2 and Z_3 respectively and so on

Parallel A.C Circuit:



Let us consider that two branch impedance $Z_1 = R_1 + jX_1$ and $Z_2 = R_2 - jX_2$

Are connected in parallel through an A.C supply voltage $V = V_m \sin \omega t$

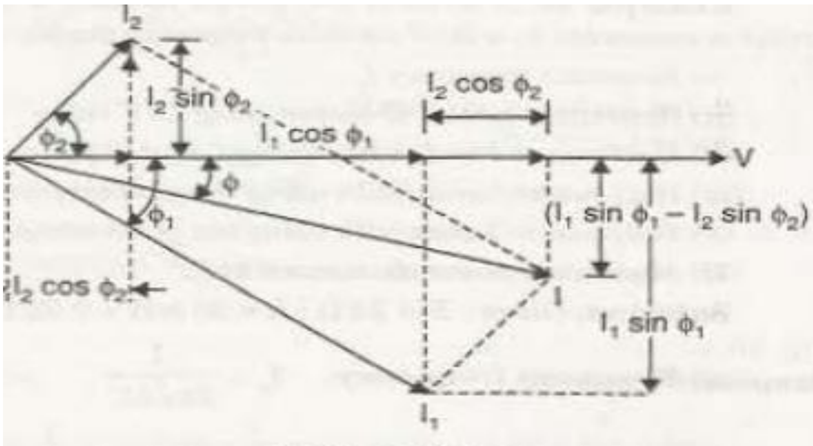
$$|Z_1| = \sqrt{R_1^{\ 2} + X_1^{\ 2}}, I_1 = \frac{V}{Z_1}, \cos\phi_1 = \frac{R_1}{Z_1}, \phi_1 = \cos^{-1}\frac{R_1}{Z_1}$$

The current I_1 is lagging the applied voltage V by an angle ϕ_1 in Z_1 branch

$$|Z_2| = \sqrt{R_2^{\ 2} + X_2^{\ 2}}\ I_2 = \frac{V}{Z_2}, \cos\phi_2 = \frac{R_2}{Z_2}, \phi_2 = \cos^{-1}\frac{R_2}{Z_2}$$

The current I_2 is lagging the applied voltage V by an angle ϕ_2 in Z_2 branch

The resultant current $\overline{I} = \overline{I_1} + \overline{I_2}$



(b) Phasor diagram

Q.1.A coils takes 2.5A, when connected across 200V,5oHz main. The power consumed by the coil is found to be 400W. Find the inductance, and power factor of the coil.

Sol: $P = I^2 R = (2.5)^2 R = 400W$

$R = \frac{400}{6.25} = 64\,\Omega$

$Z = \frac{V}{I} = \frac{200}{2.5} = 80\,\Omega$, $X_L = \sqrt{80^2 - 64^2} = 48\,\Omega$

$X_L = 2\pi * 50 * L = 48$, $L = 0.153H$

Power factor= $\cos\phi = \frac{R}{Z} = \frac{64}{80} = 0.8$ (lagging)

Q.2. An inductive coil, when connected across a 200V,50Hz supply. draws a current of 6.25A, and a power of 1000W. Another coil, connected across the same supply, draws a current of 10.75A, and a power of 1155W.Find the current drawn, and the power in net, when the two coils are connected in series across the same supply.

Sol:

For coil-1: $P_1 = 1000W = I_1^2 R_1 = (6.25)^2 R_1$

$R_1 = 25.6\,\Omega$, $Z_1 = \frac{V}{I_1} = \frac{200}{6.25} = 32\,\Omega$, $X_{L1} = \sqrt{32^2 - 25.6^2} = 19.2\,\Omega$

For Coil-2: $P_2 = 1155W = I_2^2 R_2 = (10.75)^2 R_2$, $R_2 = 10\,\Omega$

$Z_2 = \frac{V}{I_2} = \frac{200}{10.75} = 18.6\,\Omega$, $X_{L2} = \sqrt{18.6^2 - 10^2} = 15.7\,\Omega$

When two coils are connected in series , $R = 25.6 + 10 = 35.6\,\Omega$, $X_L = 19.2 + 15.7 = 34.9\,\Omega$

$Z = \sqrt{35.6^2 + 34.9^2} = 49.85\,\Omega$

$I = \frac{V}{Z} = \frac{200}{49.85} = 4.01$ A

Power= $P = I^2 R = (4.01)^2 * 35.6 = 573$ W

Q.3. Find the average power in a resistance R=10Ω if the current is

$i = 20\sin\omega t + 10\sin 3\omega t + 5\sin 5\omega t$ A.

Sol: $I_{rms} = \sqrt{\frac{1}{2}*20^2 + \frac{1}{2}*10^2 + \frac{1}{2}*5^2} = 16.2$ A

The average power $P = I^2 R = (16.2)^2 * 10 = 2624.4$ W

Q.4.A two element series circuit of R=5Ω and XL=10Ω has an effective applied voltage 100V. Determine the P, Q, S and Power factor.

$R = 5\,\Omega$ and $X_L = 10\,\Omega$

$Z = 5 + j10 = 11.18\angle 63.43^0$

$I = \frac{V}{Z} = \frac{100}{11.18\angle 63.43^0} = 8.944\angle -63.43^0$

$P = I^2 R = (8.944)^2 5 = 400$ W

$Q = I^2 X_L = (8.944)^2 10 = 800$ VA (lagging

$S = I^2 Z = (8.944)^2 11.18 = 894.4$ VA

Power factor= $\cos\phi = \frac{R}{Z} = \frac{5}{11.18} = 0.447$

3-Phase circuit:

A system with more than one phase is called polyphase system. A polyphase system contains two or more A.C voltage sources of the same frequency. These source voltages have a fixed phase angle difference between them. The most extensively used polyphase system is the 3-phase system. The three phase systems are in common use for generation, transmission, distribution and utilization of electric energy.

Advantage of 3-phase system:

1. A 3-phase machine has a smaller size as compared to 1-phase machine of the same power output.
2. The 3-phase system is used in almost all commercial electric generation.
3. The conductor material required to transmit a given power at a particular distance by 3-phase system is required less than that by a an equivalent 1-phase system.

4. For a particular size of frame, the output of a 3-phase machine is greater than of a 1-phase motor.

Generation of 3-phase supply:

The three-phase generation has three identical coils placed with their axis 120^0 apart from each other and rotated in a uniform magnetic field, a sinusoidal voltage is generated across each coil.

The sinusoidal induce voltage are

$$v_1 = v_m \sin \omega t$$

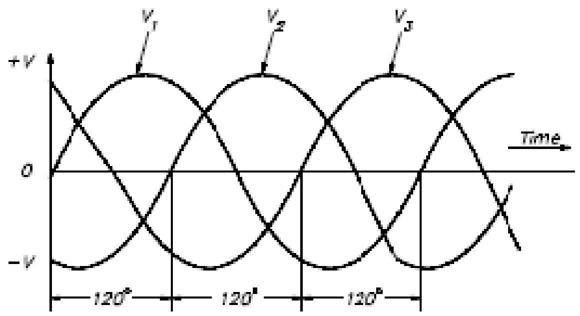
$$v_2 = v_m \sin(\omega t - 120^0)$$

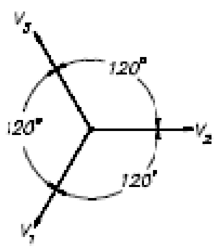
$$v_3 = v_m \sin(\omega t - 240^0)$$

$$v = v_1 + v_2 + v_3$$

$$v = v_m \sin \omega t + v_m \sin(\omega t - 120^0) + v_m \sin(\omega t - 240^0) = 0$$

Here at any instant of time, the algebraic sum of three voltage is zero.





Phase: The phase of alternating quantity implies the phase is nothing but a traction of time period that has started from reference position. The two alternating quantities are said to be in phase if they reach their zero position or reference value and maximum value at the same time. If not, they are said to be out of phase.

Phase difference:

When two alternating quantities don't reach their zero and maximum value at the same time they are said to be out of phase.

Balanced system:

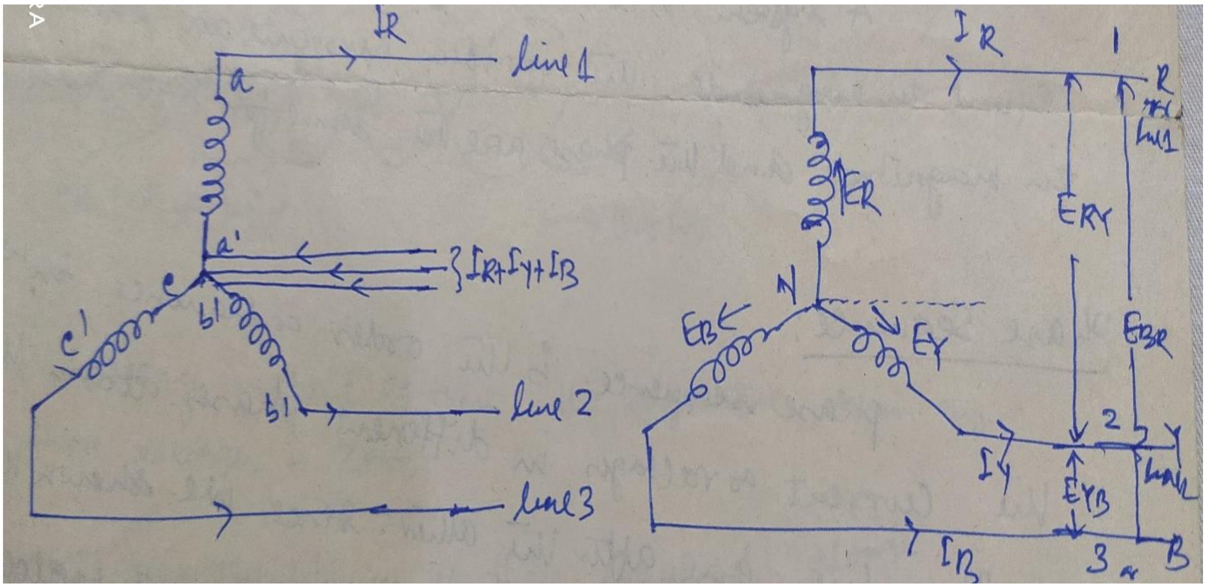
A system is balanced if various voltages are equal in magnitude, the various currents are equal in magnitude and the phase are the same for each phase.

Phase Sequence:

The phase sequence is the order or sequence in which the current or voltage in different phases attain their maximum value one after the other.

STAR CONNECTION:

As any coil having one starting end and other is finish end. In this STAR connection the three similar ends of the coils are joined together. The start ends are joined at point N and the finish ends of three windings connected to the line. The currents in each winding returns through neutral wire through the point N which is known as star point or neutral point. This type of connection is also known as 3-phase,4 wire system. The current in each winding are known as phase current but through the line is known as line current. Similarly, the voltage across each winding is known as phase voltage and voltage measured between any pair of lines or terminal are known as line voltage. The three conductors of the neutral point can be replaced by a single wire.so the summation of currents at neutral is Zero.

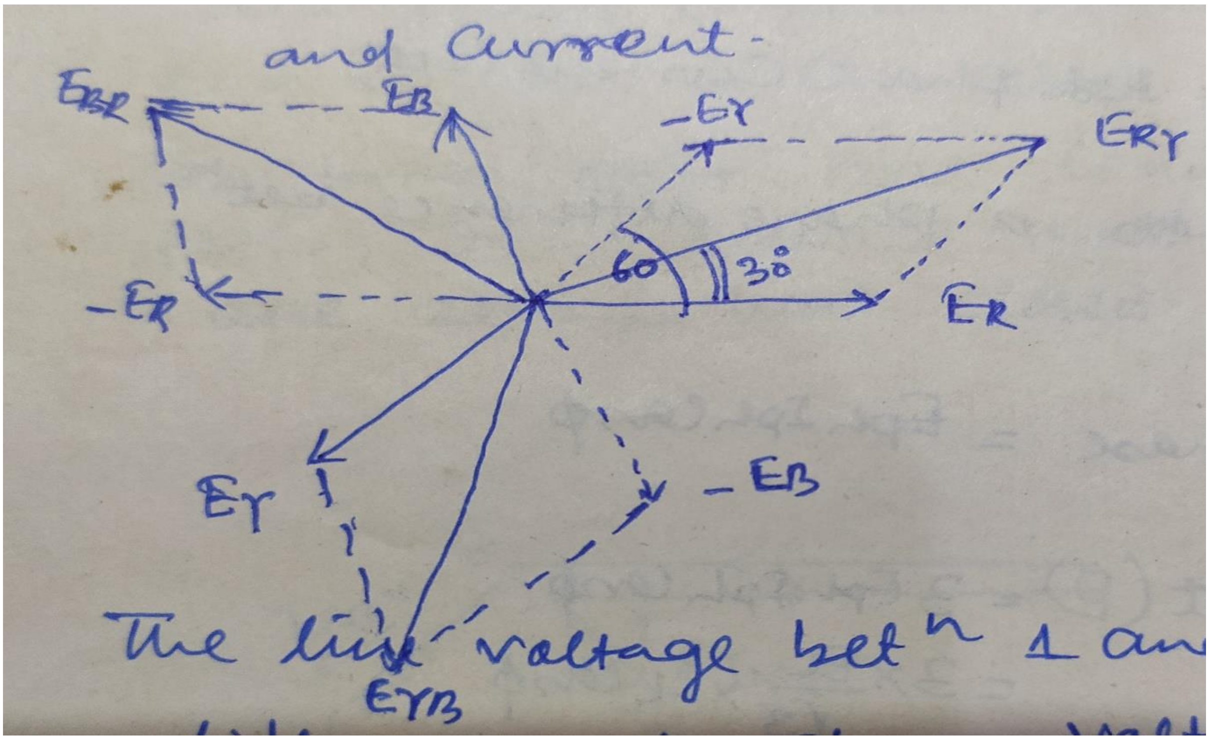


Relation between line and phase voltage.

Here the phase voltage in the Three winding are E_R, E_Y and E_B. All the phase voltages are same in magnitude but 120^0 apart as the three coils are same in all respect such type of condition is called balanced system.

$E_R = E_Y = E_B = E_{ph}$ (phase voltage)

Similarly the voltage available between any pair of terminals is called line voltage (E_{RY}, E_{BR}, E_{YB}) the current flowing in each lines are I_R, I_Y and I_B, it is called line current.



The line voltage between 1 and 2 or line voltage E_{RY} is the vector difference of phase voltages E_R and E_Y.

$E_{RY} = E_R - E_Y$ (Vector difference)

$E_{RY} = E_R + (-E_Y)$ (Vector sum)

Since the phase angle between vectors E_R and ($-E_Y$) is 60^0

From the vector diagram $E_{RY} = \sqrt{|E_R|^2 + |E_Y|^2 + 2|E_R||-E_Y|\cos 60^0}$

But ($E_R = E_Y = E_B = E_{ph}$)

$$E_{RY} = \sqrt{E^2{}_{ph} + E^2{}_{ph} + 2E_{ph}E_{ph}\frac{1}{2}}$$

$$E_{RY} = \sqrt{3}E_{ph}$$

Similarly, line voltages between 2 and 3

$$E_{YB} = E_Y - E_B = \sqrt{3}E_{ph}$$

And line voltage between 3 and 1

$$E_{BR} = E_B - E_R = \sqrt{3}E_{ph}$$

Line voltage $E_L = \sqrt{3}E_{ph}$

Relation between line current and phase current.

$I_R = I_Y = I_B = I_{ph}$

Line current $I_L = I_{ph}$ (phase current)

Power:

If the three-phase current has a phase differences between the phase voltage is ϕ

Then power output per phase= $E_{ph} I_{ph} \cos\phi$

Total power output $P = 3E_{ph} I_{ph} \cos\phi$

$$P = 3\frac{E_L}{\sqrt{3}} I_L \cos\phi$$

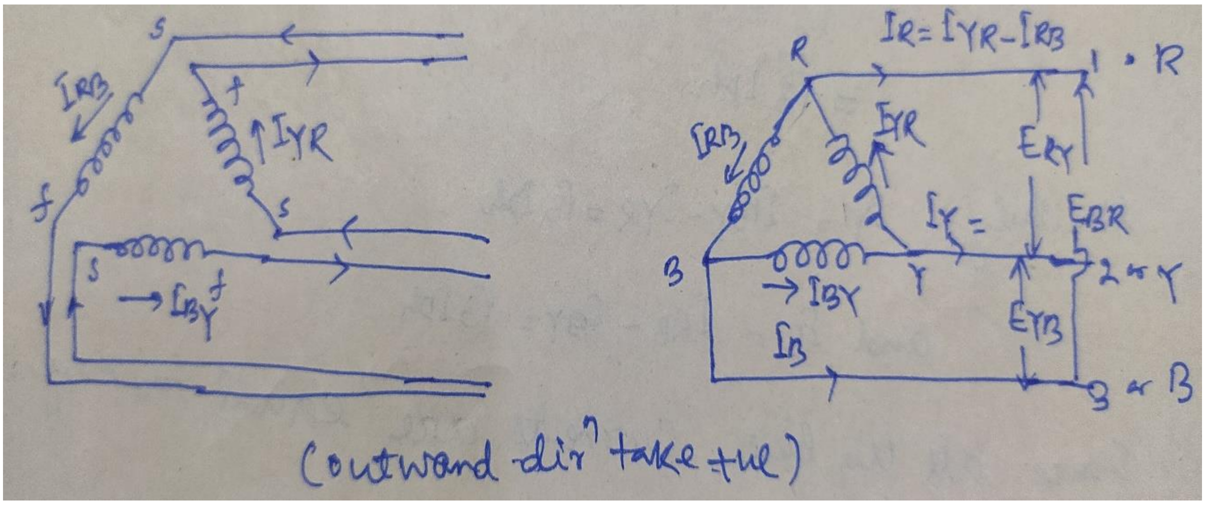
$P = \sqrt{3} E_L I_L \cos\phi$ = Total active power

Total reactive power $Q = \sqrt{3} E_L I_L \sin\phi$

Total apparent power $S = \sqrt{3} E_L I_L$

DELTA CONNECTION:

In Delta connection, the dissimilar ends of the coil are joined together in triangular form that means the starting end of one is connected with the finishing end of another. The three coils are connected in series and formed a closed path. It is clear that the summation of voltages is Zero in that closed path as the system become balanced. Here outward direction taken as positive.



Relation between line current and phase current

It is clear that line current is vector difference of phase current of two-phase current.

Line current in line R is $I_R = I_{YR} - I_{RB}$ (Vector difference)

$$I_R = I_{YR} + (-I_{RB}) \text{ (Vector Sum)}$$

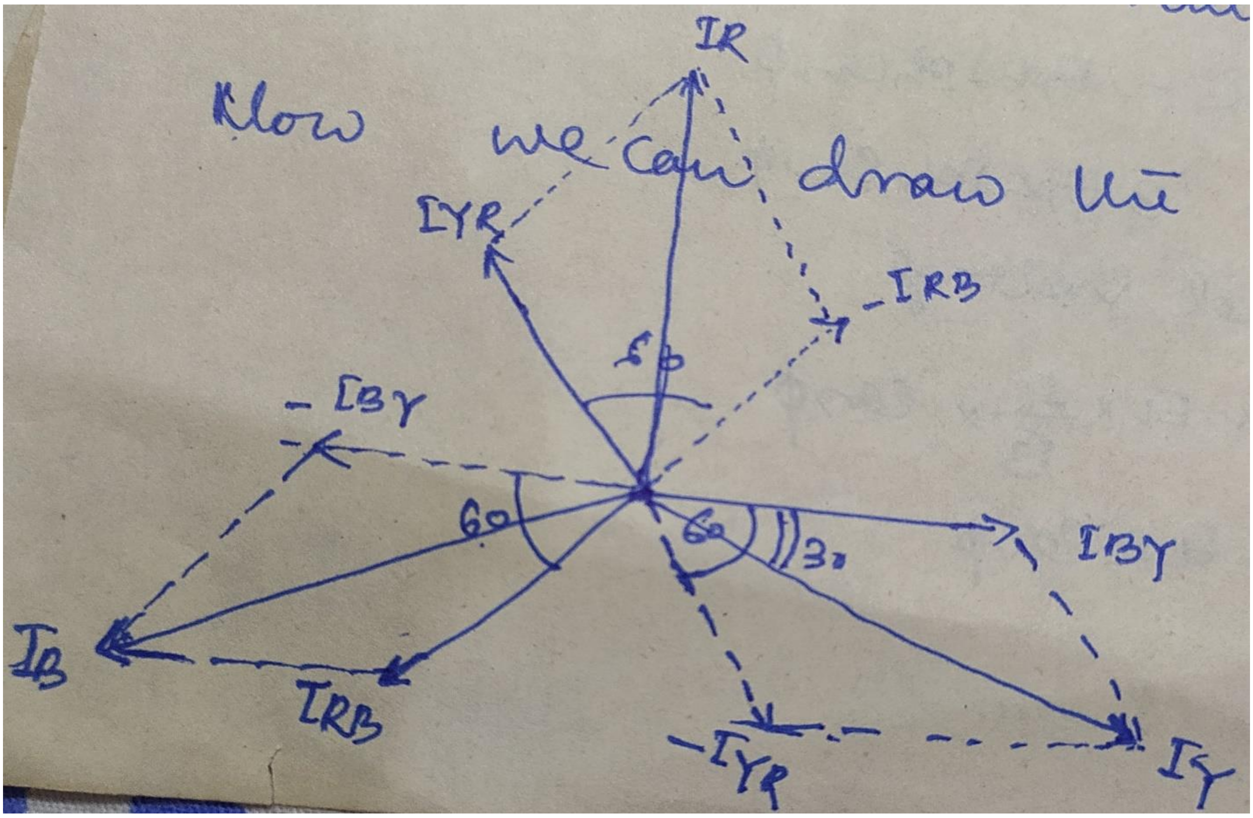
Where I_{RB} and I_{YR} are phase current.

Similarly line current in line Y is $I_Y = I_{BY} - I_{YR}$

$$I_Y = I_{BY} + (-I_{YR})$$

And line current in line B is $I_B = I_{RB} - I_{BY}$

$$I_B = I_{RB} + (-I_{BY})$$



Vector diagram

Since phase angle between phase current I_{YR} and $-I_{RB}$ is 60^0

$$I_R = \sqrt{|I_{YR}|^2 + |I_{RB}|^2 + 2|I_{YR}||-I_{RB}|\cos 60^0}$$

$$I_R = \sqrt{I^2{}_{ph} + I^2{}_{ph} + 2I_{ph}I_{ph}\frac{1}{2}} \text{ as } (I_{YR} = I_{RB} = I_{BY} = I_{ph})$$

$$I_R = \sqrt{3}I_{ph}$$

Similarly, $I_Y = I_{BY} - I_{YR} = \sqrt{3}I_{ph}$ and $I_B = I_{RB} - I_{BY} = \sqrt{3}I_{ph}$

$$I_R = I_Y = I_B = I_L$$

$$I_L = \sqrt{3} I_{ph}$$

Relation between line voltage and phase voltage

The voltage between any pair of line is equal to the phase voltage of the phase winding connected between the two lines considered.

$$\text{Line voltage } (E_L) = \text{Phase voltage } (E_{ph})$$

Power:

Then power output per phase= $E_{ph} I_{ph} \cos\phi$

Total power output $P = 3E_{ph} I_{ph} \cos\phi$

In term of line voltage and current

Total power $P = 3E_L \frac{I_L}{\sqrt{3}} \cos\phi$

$$P = \sqrt{3} E_L I_L \cos\phi$$

Q.1.A balanced star connected load of (8+j6) Ω per phase is connected to a three phase 230V supply. Find the line current, power factor, active power and total volt-amps.

$Z_{ph} = (8 + j6)\,\Omega = 10\angle 36.87^0, V_L = 230$ V

For star connection, $E_L = \sqrt{3} E_{ph}, V_{ph} = \frac{230}{\sqrt{3}} = 132.79$ V

(i) $I_L = I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{132.79}{10\angle 36.87} = 13.28\angle -36.87^0$ A

(ii) power factor=cos36.87^0=0.8(lagging)

(iii)Active power= $P = \sqrt{3} E_L I_L \cos\phi = \sqrt{3} * 230 * 13.28 * 0.8 = 4232$W

(iv) Reactive power= $Q = \sqrt{3} E_L I_L \sin\phi = \sqrt{3} * 230 * 13.28 * 0.6 = 3174$Var

(v)Total VA= $S = \sqrt{3} E_L I_L = \sqrt{3} * 230 * 13.28 = 5290$VA

Q.2.Three similar coils, each of resistances 20Ω, and inductance 0.5H, are connected in Delta to a 3-phase,50Hz,400V supply, calculate the line current, and total power absorbed.

$R_{ph} = 20\,\Omega,\ X_L = 2\pi * 50 * 0.5 = 157\Omega$

$|Z_{ph}| = \sqrt{20^2 + 157^2} = 158.3\Omega,\ \cos\phi = \frac{20}{158.3} = 0.1264$(lag)

$V_{ph} = V_L = 400$ V

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{158.3} = 2.528\,\text{A},$$

$$I_L = \sqrt{3} I_{ph} = \sqrt{3} * 2.258 = 4.38\ \text{A}$$

$$P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} * 400 * 4.38 * 0.1264 = 383.6W$$

Coupled Circuit:

The coupled circuit associated with magnetic circuit. The best example is 1-phase transformer. In single phase transformer having two circuit or winding they are electrically isolated with each other but magnetically coupled with each other. In this circuit both self and mutual inductance present for the operation of transformer by supplying an alternating voltage to the circuit/winding.

Self-Inductance:

As per Faraday's Law, when a current change in a circuit, the magnetic flux linking the same circuit changes and an emf is induced in the circuit.

This induced emf in the circuit is proportional to the rate of change of current.

$V_L = L\frac{dI}{dt}$ where

L is the self-inductance of the circuit

Also, L= $\frac{N\phi}{I}$

N=Number of turns in the circuit, φ=flux linkage of the circuit

VL=L $\frac{d\frac{N\phi}{L}}{dt}$ =N $\frac{d\phi}{dt}$

Mutual Inductance:

When two coils are electrically isolated but magnetically coupled with each other than there will be mutual inductance present between these coils.

Suppose I_1 and I_2 are the two-current carrying in the coil-1 and coil-2 respectively than due to these current produces' leakage flux as well as linkage flux or mutual flux.

Let ϕ_{11} and ϕ_{22} are the leakage flus in coil-1 and coil-2

ϕ_{21} and ϕ_{12} are linkage or mutual fluxes in coil-1 and coil-2 respectively.

When current i1 is passes through the coil-1 than voltage is induced in the coil-2 which is given by

$$V_{L2} = N_2 \frac{d\phi 12}{dt} \qquad (1)$$

As I1 is directly proportional to $\phi_{12} N_2$

Where N_2 =No. of turn in coil-2

$\phi_{12} N_2 = MI_1$ where M is constant of proportionality called mutual inductance between the two coil.

$$V_{L2} = \frac{d(N2\phi 12)}{dt} = \frac{d(MI1)}{dt}$$

$$V_{L2} = M \frac{dI1}{dt} \qquad (2)$$

From equation (1) and (2),

$$N2 \frac{d\phi 12}{dt} = M \frac{dI1}{dt}$$

$$M = N2 \frac{d\phi 12}{dI1} \qquad (3)$$

Similarly, when current passes through the coil-2 than mutual inductance is given by

$$M = N1 \frac{d\phi 21}{dI2}$$

Co-efficient of coupling (k):

The fraction of total flus which links the coil is called the co-efficient of coupling(k)

K= $\frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$ where $\phi_1 = \phi_{11} + \phi_{12}$ $\phi_2 = \phi_{22} + \phi_{21}$ $\phi 1 \geq \phi_{12}, \phi_2 \geq \phi_{21}$

As M=N2 $\frac{d\phi 12}{dI1}$ (1)

M=N1 $\frac{d\phi 21}{dI2}$ (2)

Multiplying equation (1) and (2)

$$M^2 = \frac{N_1 N_2 \phi_{12} \phi_{21}}{I_1 I_2}$$

$$M^2 = \frac{N_1 N_2 k\phi_1 k\phi_2}{I_1 I_2} = k^2 \frac{N_1\phi_1}{I_1} \frac{N_2\phi_2}{I_2} = k^2 L_1 L_2 \text{ As } L_1 = \frac{N_1\phi_1}{I_1} \text{ and } L_2 = \frac{N_2\phi_2}{I_2}$$

$$M^2 = k^2 L_1 L_2$$

$$M = k\sqrt{L_1 L_2}$$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

This constant k is called the co-efficient of coupling.

It is also defined as the ratio of mutual inductance actually present between the two coils to the maximum possible value of mutual inductance.

The maximum possible value of mutual inductance can be obtained in three different cases

Case-1: If the flux due to one coil completely links with other than k=1(Coil tightly coupled)

Case-2: If the flux due to one coil dose not link with other than k=0(magnetically isolated from each other)

Case-3: If flux due to one coil slightly /partially links with other coil than k has finite value(0<K>1)

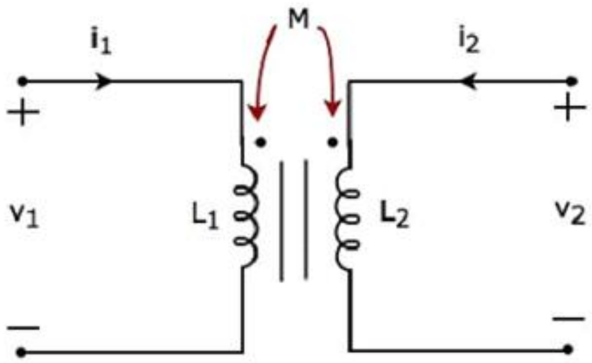
DOT Convention:

Supposed in the given coupled circuit coil-1 and coil-2 having N1 and N1 turns carrying current i1 and i2 produces two voltage drop in each coil (one voltage produces due to self-inductance and other voltage produces due to mutual inductance. The voltage induced due to mutual inductance is either positive or negative sign depend upon the direction of flow of fluxes or can be determined by use of DOT convention. For this we can apply DOT Rule

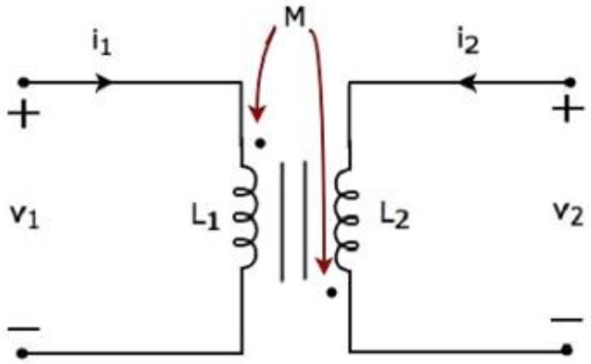
DOT Rule:

(1) When both assumed currents enter or leave a pair of coupled coils at the dotted terminals, the sign of the M terms will be the same as the sign of the L terms

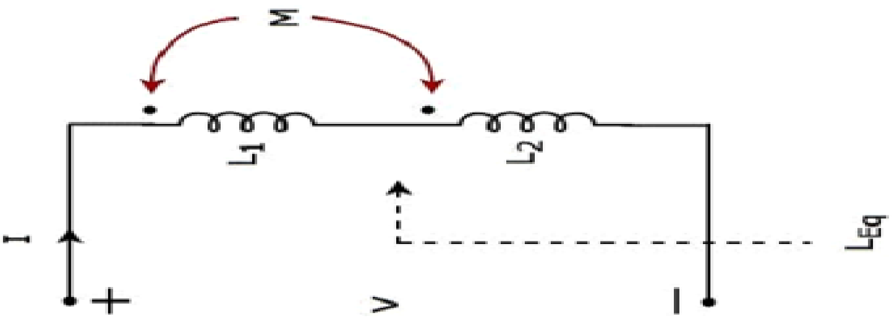
(2) When one current enters at a dotted terminal and leaves by a dotted terminal, the sign of the M terms is opposite to the sign of the L terms.



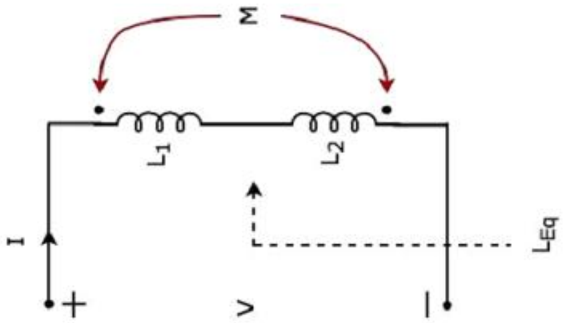
(+M)



(-M)

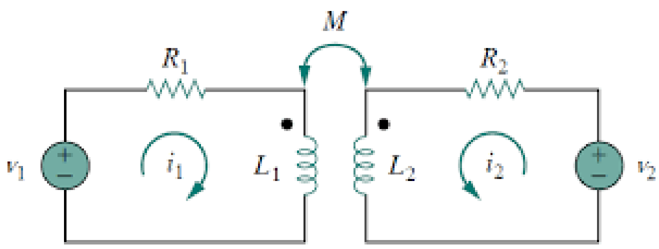


(+M)



(-M)

Voltage Equation of two coupled coils (time domain and frequency domain)



The above coupled circuit we can express the voltage equation in term of time domain and frequency domain

$$V_1 - i_1R_1 - L_1\frac{di_1}{dt} - [+M\frac{di_2}{dt}] = 0 \qquad (1)$$

$$V_2 - i_2R_2 - L_2\frac{di_2}{dt} - [+M\frac{di_1}{dt}] = 0 \qquad (2)$$

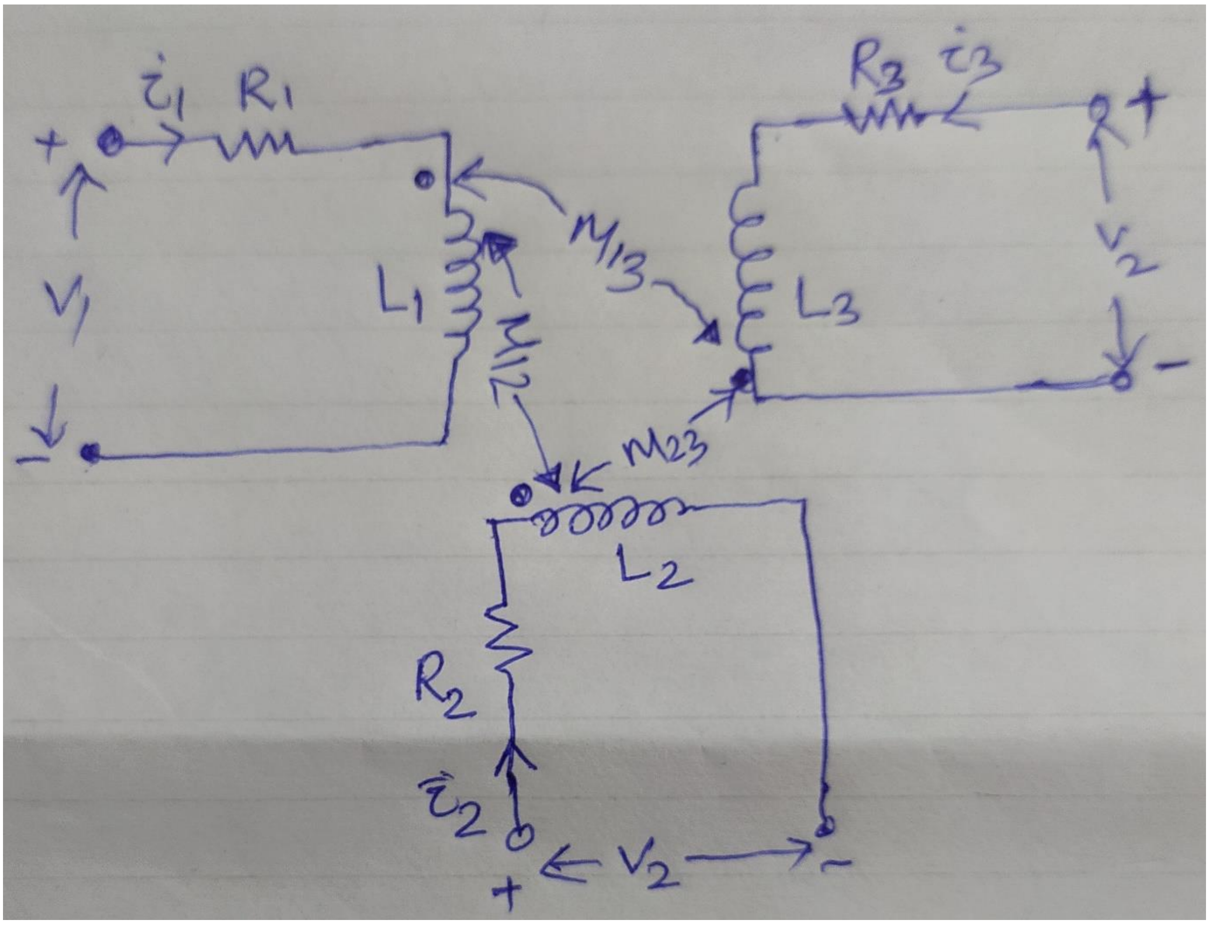
The equation-1 and 2 known as time domain voltage equation of coupled circuit.

The voltage equation-1 and equation-2 can be expresses in term of frequency domain as follows

$$V_1 - i_1R_1 - j\omega L_1 i_1 - j\omega M i_2 = 0$$

$$V_2 - i_2R_2 - j\omega L_2 i_2 - j\omega M i_1 = 0$$

Voltage Equation of three coupled coils.



$$V_1 - i_1R_1 - L_1\frac{di_1}{dt} - [+M_{12}\frac{di_2}{dt}] - [-M_{13}\frac{di_3}{dt}] = 0 \qquad (1)$$

$$V_2 - i_2R_2 - L_2\frac{di_2}{dt} - [+M_{12}\frac{di_1}{dt}] - [-M_{23}\frac{di_3}{dt}] = 0 \qquad (2)$$

$$V_3 - i_3R_3 - L_3\frac{di_3}{dt} - [M_{13}\frac{di_1}{dt}] - [-M_{23}\frac{di_2}{dt}] = 0 \qquad (3)$$

The equation 1,2 and 3 are voltage equation of time domain

$$V_1 - i_1R_1 - j\omega L_1 i_1 - j\omega M_{12} i_2 + j\omega M_{13} i_3 = 0 \qquad (4)$$

$$V_2 - i_2R_2 - j\omega L_2 i_2 - j\omega M_{12} i_1 + j\omega M_{23} i_3 = 0 \qquad (5)$$

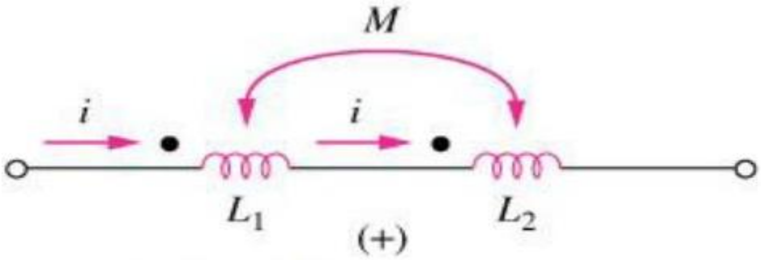
$$V_3 - i_3R_3 - j\omega L_3 i_3 - j\omega M_{13} i_1 + j\omega M_{23} i_2 = 0 \qquad (6)$$

The equation 4,5 and 6 are the voltage equation in term of frequency domain.

Different connection of coupled coils;

(a)**Series connection of coupled coils:(Series additive)**

When two coupled coils are connected in series with dot is given entering point of both the coils.



Then from the above circuit total voltage V=VL1+VL2

$$V_1 = L_1\frac{di}{dt} + M\frac{di}{dt}$$

$$V_2 = L_1\frac{di}{dt} + M\frac{di}{dt}$$

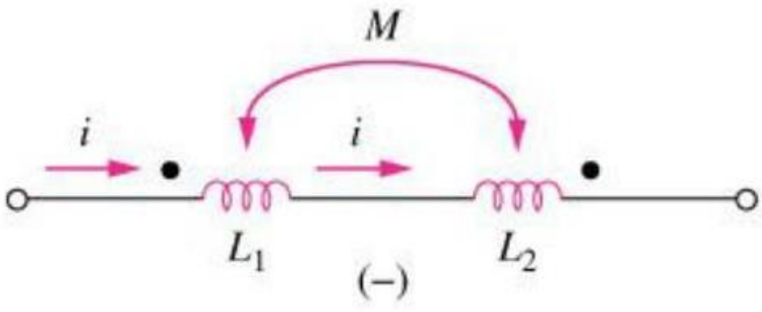
$$V = L_1\frac{di}{dt} + M\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt}$$

$$V = (L_1 + L_2 + 2M)\frac{di}{dt}$$

$$L_{eq}\frac{di}{dt} = (L_1 + L_2 + 2M)\frac{di}{dt}$$

$L_{eq} = (L_1 + L_2 + 2M)$ Equivalent inductance in series connection of two coupled coil in series additive

Csae-2: Series connection of coupled coil (series opposing)



V=VL1+VL2

$$V_1 = L_1 \frac{di}{dt} - M\frac{di}{dt}$$

$$V_2 = L_1 \frac{di}{dt} - M\frac{di}{dt}$$

$$V = L_1 \frac{di}{dt} - M\frac{di}{dt} + L_2\frac{di}{dt} - M\frac{di}{dt}$$

$$V = (L_1 + L_2 - 2M)\frac{di}{dt}$$

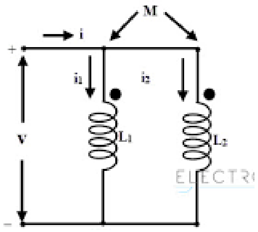
$$L_{eq}\frac{di}{dt} = (L_1 + L_2 - 2M)\frac{di}{dt}$$

$L_{eq} = (L_1 + L_2 - 2M)$ Equivalent inductance in series connection of two coupled coil in series opposing

Case-3: Parallel connection of coupled coils

$i = i_1 + i_2$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \qquad (1)$$



$$V = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \qquad (2)$$

$$V = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \qquad (3)$$

$$L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$\frac{di_1}{dt} = (\frac{L_2 - M}{L_1 - M}) \frac{di_2}{dt} \qquad (4)$$

Putting the value of $\frac{di_1}{dt}$ in equation (1)

$$\frac{di}{dt} = (\frac{L_2 - M}{L_1 - M}) \frac{di_2}{dt} + \frac{di_2}{dt} \quad ,$$

$$\frac{di}{dt} = [\frac{L_2 - M}{L_1 - M} + 1] \frac{di_2}{dt} \qquad (5)$$

From equation-2

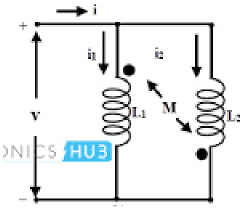
$$L \frac{di}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$\frac{di}{dt} = \frac{1}{L} [L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$[\frac{L_2 - M}{L_1 - M} + 1] \frac{di_2}{dt} = \frac{1}{L} [L_1 (\frac{L_2 - M}{L_1 - M}) + M] \frac{di_2}{dt}$$

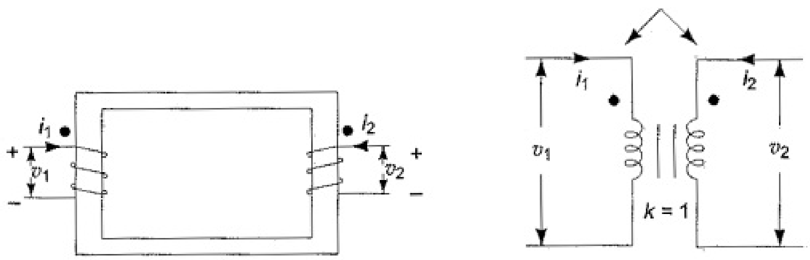
$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \qquad \text{(Parallel adding)}$$

Similarly, in the same brochure for parallel opposing the equivalent inductance



$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

Ideal Transformer



(a) (b) $N_1 : N_2$

The magnitude of self induced emf $V = L\frac{di}{dt} = N\frac{d\phi}{dt}$

$$\Rightarrow L = N\frac{d\phi}{di}$$

But $\phi = \frac{Ni}{S}$,S= reluctance

$$L = N\frac{d}{di}(\frac{Ni}{S})$$

$$L = N\frac{d}{di}(\frac{Ni}{S}) = \frac{N^2}{S}$$

From the above $\frac{L_2}{L_1} = \frac{{N_2}^2}{{N_1}^2} = a^2$

The ideal transformer is very useful model for circuit calculations. The input impedance of the transformer can be determined by using the voltage equation in (b) by inserting a load impedance Z_L in secondary side coil across v_2 .

$$V_1 = j\omega L_1 I_1 - j\omega M I_2 \qquad (1)$$

$$0 = -j\omega M I_1 + (Z_L + j\omega L_2) I_2 \qquad (2)$$

Where V1 and V2 are thr voltage phasors and I1 ,I2 are the current phasors

From equation (2) , $I_2 = \frac{j\omega M I_1}{(Z_L + j\omega L_2)}$

Put I2 in equation (1)

$$V_1 = j\omega L_1 I_1 + \frac{I_1 \omega^2 M^2}{Z_L + j\omega L_2}$$

The input impedance $Z_{in} = \frac{V_1}{I_1} = j\omega L_1 + \frac{\omega^2 M^2}{(Z_L + j\omega L_2)}$

Let us assumed k=1, $M = \sqrt{L_1 L_2}$

$$Z_{in} = \frac{V_1}{I_1} = j\omega L_1 + \frac{\omega^2 L_1 L_2}{(Z_L + j\omega L_2)}$$

We have already known the relation, $\frac{L_2}{L_1} = a^2$ where $a = \frac{N_2}{N_1}$ =turn ratio

$$Z_{in} = j\omega L_1 + \frac{\omega^2 {L_1}^2 a^2}{(Z_L + j\omega L_2)}$$

$$Z_{in} = \frac{(Z_L + j\omega L_2) j\omega L_1 + \omega^2 {L_1}^2 a^2}{Z_L + j\omega L_2}$$

$$Z_{in} = \frac{j\omega L_1 Z_L}{Z_L + j\omega L_2}$$

As L_2 is assumed to be infinitely large compared to Z_L

$$Z_{in} = \frac{j\omega L_1 Z_L}{j\omega a^2 L_1} = \frac{Z_L}{a^2} = (\frac{N_1}{N_2})^2 Z_L$$

Q.1.In a pair of coupled coils coil 1 has a continuous current of 2A, and the corresponding fluxes ϕ_{11} and ϕ_{21} are 0.3 and 0.6mwb respectively. If the turns are N_1 =500 and N_2 =1500,find L_1, L_2, M and k .

Sol:

Total flux $\phi_1 = \phi_{11} + \phi_{21} = 0.3 + 0.6 = 0.9$ mwb

$$L_1 = \frac{N_1 \phi}{i_1} = \frac{(500)(0.9 \times 10^{-3})}{2} = 0.225H$$

$$k = \frac{\phi_{21}}{\phi_1} = \frac{0.6}{0.9} = 0.667$$

$$M = \frac{N_2 \phi_{21}}{i_1} = \frac{(1500)(0.6 \times 10^{-3})}{2} = 0.45H$$

$$M = k\sqrt{L_1 L_2}$$

$$0.45 = 0.667\sqrt{(0.225)L_2}$$

$$L_2 = 2.023H$$

Q.2. Two coupled coils with L1=0.01H and L2=0.04H and k=0.6 are connected in four different ways, series aiding, series opposing, parallel aiding and parallel opposing. Find the equivalent inductances in each case.

Sol: L_1 =0.01H, L_2 =0.04H and k =0.6

$M = k\sqrt{L_1L_2} = 0.6\sqrt{(0.01)(0.04)} = 0.012H$

(i) Series aiding $L_{eq} = (L_1 + L_2 + 2M) = 0.01 + 0.04 + 2(0.012) = 0.074H$

(ii) Series opposing $L_{eq} = (L_1 + L_2 - 2M) = 0.01 + 0.04 - 2(0.012) = 0.026H$

(iii) Parallel aiding

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} = \frac{(0.01)(0.04) - (0.012)^2}{(0.01) + (0.04) - 2(0.012)} = 9.846mH$$

(iv)Parallel opposing $L_{eq} = \dfrac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \dfrac{(0.01)(0.04) - (0.012)^2}{0.01 + 0.04 + 2(0.012)} = 3.459mH$

Q.3. Two coils when connected in series have a combined inductance of 0.8H or0.6H depending on the mode of connection, one of the coils when isolated from the other has inductance of 0.3H. Find

(a) The mutual inductance between the two coils

(b) the inductance of the other coil

(c) the coupling co-efficient

Sol: $0.8 = (L_1 + L_2 + 2M)$ (1)

$0.6 = (L_1 + L_2 - 2M)$ (2)

Adding equation (1) and (2)

$1.4 = 2(L_1 + L_2)$

$L_2 = 0.7 - 0.3 = 0.4H$

From equation (1), $0.8 = (0.3 + 0.4 + 2M)$

$M = 0.05H$

$M = k\sqrt{L_1L_2}$

$0.05 = k\sqrt{0.3 * 0.4}, k = 0.144$