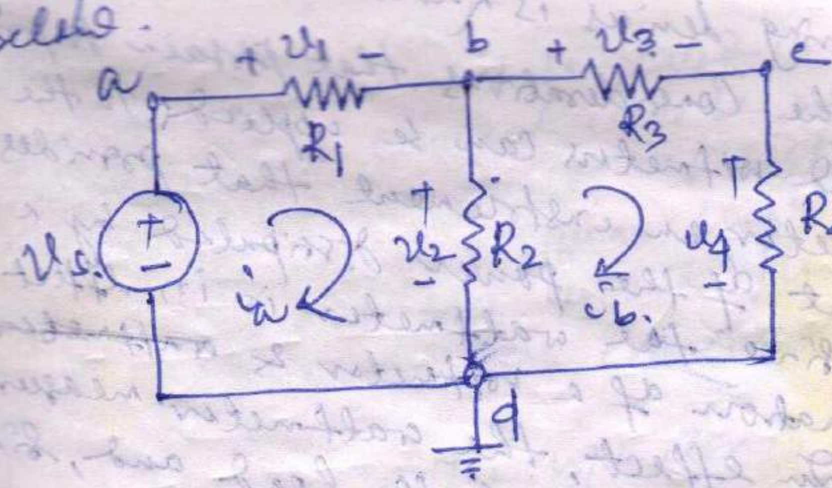
Resistive Network Analysis :→

The analysis of an electric network consists of determining each of the unknown branch currents and node voltages. This is done by writing the smallest set of equations sufficient to solve for all network variables. For this it is important to define all the relevant variables in systematic manner. The following example demonstrates how to define all the voltages and currents that are associated with a specific circuit.

Ex→3.1 / R-82 Identify the branch and node voltages and the loop and mesh currents in the circuit of fig. given below.



Sol:→ The following node voltages may be identified & the branch voltages can be ~~identified~~ obtained accordingly.

Node Voltages	Branch voltages
$v_a = v_s$ (source voltage)	$v_s = v_a - v_d = v_a$
$v_b = v_2$	$v_1 = v_a - v_b$
$v_c = v_4$	$v_2 = v_b - v_d = v_b$
$v_d = 0$ (ground)	$v_3 = v_b - v_c$
	$v_4 = v_c - v_d = v_c$

It is desirable to reduce the number of equations needed to solve a circuit to the minimum necessary, that is, a method for obtaining N equations in N unknowns.

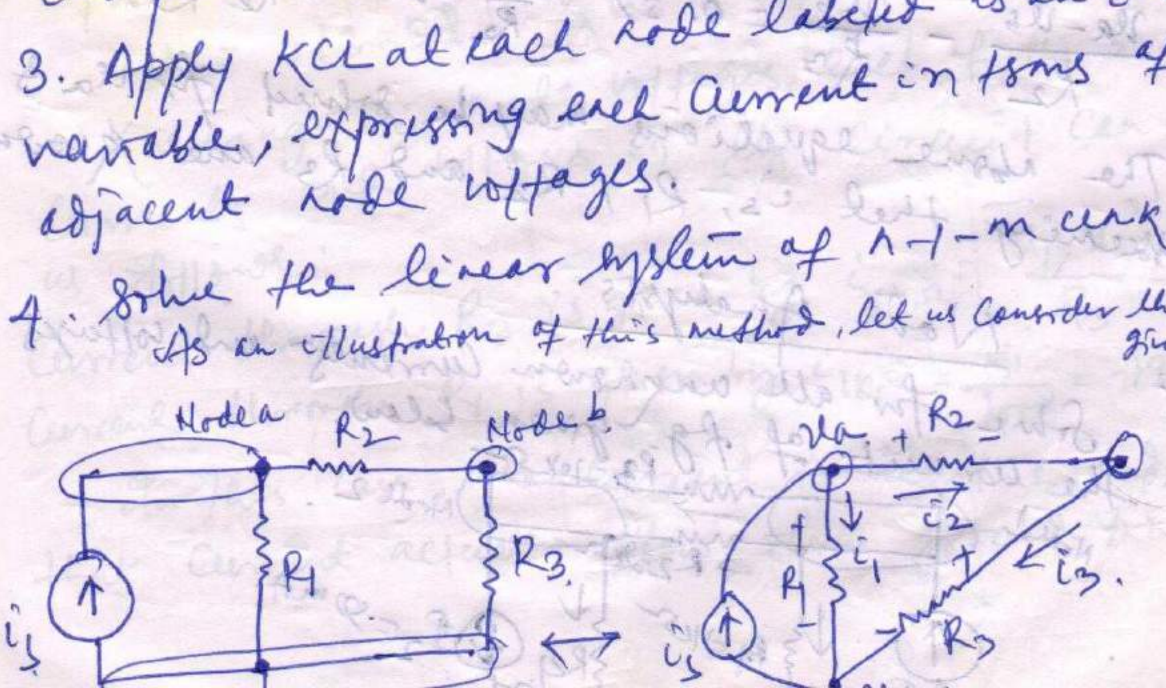
THE NODE VOLTAGE METHOD :→

The node voltage method is based on defining the voltage at each node as an independent variable. One of the nodes is selected as a reference node (usually - but not necessarily - ground), and each of the other node voltages is referenced to this node. In a circuit containing n nodes, we can write at most (n-1) independent equations.

STEPS :→

1. Select a reference node (usually ground). This node has most elements tied to it. All other nodes are referenced to this node.
2. Define the remaining (n-1) node voltages as the independent or dependent variables. Each of the m voltage sources in the circuit is associated with a dependent variable. If a node is not connected to a voltage source, then its voltage is treated as an independent variable.
3. Apply KCL at each node labeled as an independent variable, expressing each current in terms of the adjacent node voltages.
4. Solve the linear system of n-1-m unknowns.

As an illustration of this method, let us consider the circuit given below.



The direction of current ...
(assuming that i_s is a positive current).

Application of KCL at node 'a' yields

$i_s - i_1 - i_2 = 0.$

and at node b

$i_2 - i_3 = 0.$

It should be noted that it is not necessary to apply KCL at the reference node. The equation obtained at node 'c' is $i_1 + i_3 - i_s = 0$ is not independent of the above equations & in fact, it may be obtained by adding the equations obtained at node 'a' & node 'b'. This observation confirms the statement that in a circuit containing n nodes, we can write at most (n-1) independent equations.

Now, the above two equations can be written in terms of node voltage as follows:

$$i_s - \frac{v_a}{R_1} - \frac{v_a - v_b}{R_2} = 0 \Rightarrow \left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_a - \frac{1}{R_2}v_b = i_s$$

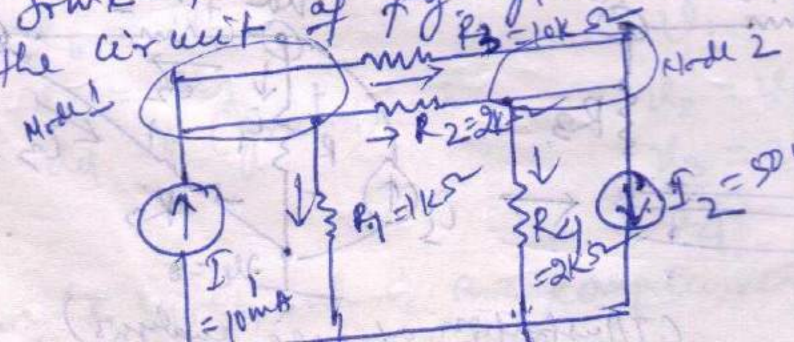
$$\frac{v_a - v_b}{R_2} - \frac{v_b}{R_3} = 0 \Rightarrow -\frac{1}{R_2}v_a + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)v_b = 0.$$

The above equations may be solved for v_a & v_b assuming that i_s, R_1, R_2 and R_3 are known.

Node Analysis

Ex:-3.2. R-85

Solve for all unknown currents and voltages in the circuit of fig. given below.



Sol: Applying KCL at nodes 1 & 2, we obtain.

$$I_1 - \frac{v_1 - 0}{R_1} - \frac{v_1 - v_2}{R_2} - \frac{v_1 - v_2}{R_3} = 0 \quad \text{node } 1$$

$$\frac{v_1 - v_2}{R_2} + \frac{v_1 - v_2}{R_3} - \frac{v_2 - 0}{R_4} - I_2 = 0 \quad \text{node } 2.$$

The above two equations can be written more systematically as a function of the unknown node voltages as follows:

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) v_1 + \left(-\frac{1}{R_2} - \frac{1}{R_3}\right) v_2 = I_1 \quad \text{node } 1$$

$$\left(-\frac{1}{R_2} - \frac{1}{R_3}\right) v_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) v_2 = -I_2 \quad \text{node } 2$$

Putting the values of R_1, R_2 & R_3 in the above eq^ns, finally we get:

$$1.6 v_1 - 0.6 v_2 = 10$$

$$-0.6 v_1 + 1.1 v_2 = -50.$$

The solutions of the above equation is:

$v_1 = -13.57$ V. & $v_2 = -52.86$ V.

Knowing the node voltages, each of the branch currents and voltages in the circuit can be determined as follows:

Current through R_3 is given by $i_{10k\Omega} = \frac{v_1 - v_2}{R_3} = 3.93$ mA.

Current through R_1 is given by $i_{1k\Omega} = \frac{v_1}{1000} = -13.57$ mA.

In this case, the current is negative, indicating that current actually flows from ground to node 1.

SOLUTION OF LINEAR SYSTEM OF EQUATIONS USING CRAMER'S RULE:→

Let us consider the following linear system of eqns where x_1, x_2 & x_3 are unknowns.

$$R_{11}x_1 + R_{12}x_2 + R_{13}x_3 = K_1$$

$$R_{21}x_1 + R_{22}x_2 + R_{23}x_3 = K_2$$

$$R_{31}x_1 + R_{32}x_2 + R_{33}x_3 = K_3$$

The above equation can be written in matrix form as follows:

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix}$$

Now, the solution of the above equations can be obtained as follows:

$$x_1 = \frac{\Delta_1}{\Delta}, \quad x_2 = \frac{\Delta_2}{\Delta} \quad \& \quad x_3 = \frac{\Delta_3}{\Delta}$$

where $\Delta = \begin{vmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{vmatrix}$

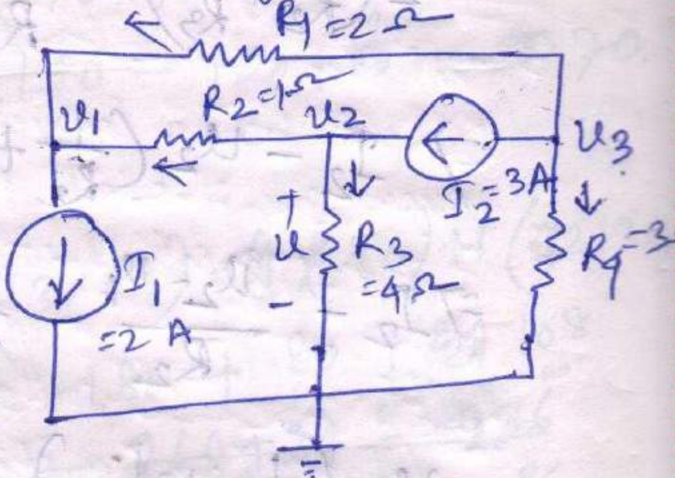
$$\Delta_1 = \begin{vmatrix} K_1 & R_{12} & R_{13} \\ K_2 & R_{22} & R_{23} \\ K_3 & R_{32} & R_{33} \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} R_{11} & K_1 & R_{13} \\ R_{21} & K_2 & R_{23} \\ R_{31} & K_3 & R_{33} \end{vmatrix}$$

$| R_{11} \quad R_{12} \quad K_1 |$

Ex: 3.5
R-89

Use the node voltage analysis to determine the voltage v in the circuit of fig. given below.



Sol:→ Applying KCL at the three nodes we have:

$$\frac{v_3-v_1}{R_1}+\frac{v_2-v_1}{R_2}-I_1=0 \text{ for node 1}$$

$$I_2-\frac{v_2-v_1}{R_2}-\frac{v_2}{R_3}=0$$

$$-\frac{\cancel{v_1-v_3}\ v_3-v_1}{R_1}-\frac{v_3}{R_4}-I_2=0$$

Putting the values of R_1, R_2, R_3 & R_4 and I_1 & I_2 the above equation can be written as:

$$\frac{v_3-v_1}{2}+\frac{v_2-v_1}{1}-2=0 \Rightarrow v_3-v_1+2v_2-2v_1=4$$

$$\Rightarrow -3v_1+2v_2+v_3=4$$

$$3-\frac{v_2-v_1}{1}-\frac{v_2}{4}=0 \Rightarrow 4v_2-4v_1+v_2=12.$$

$$\Rightarrow -4v_1+5v_2=12$$

$$\cancel{\Rightarrow 4v_1+8v_2=12.}$$

$$-\frac{v_3-v_1}{2}-\frac{v_3}{3}-3=0 \Rightarrow -18=3v_3-3v_1+2v_3=0$$

$$\Rightarrow -3v_1+5v_3=-18$$

The above three equations can also be written by direct inspection as follows:

$$v_1\left(\frac{1}{R_1}+\frac{1}{R_2}\right)-\frac{v_2}{R_2}-\frac{v_3}{R_1}=-I_1 \quad \underline{\text{For node 1}}$$

$$\Rightarrow \frac{v_1-v_3}{R_1}+\frac{v_1-v_2}{R_2}+I_1=0.$$

$$v_3-v_1 \quad -I_1=0.$$

$$v_2\left(\frac{1}{R_2}+\frac{1}{R_3}\right)-\frac{v_1}{R_2}=I_2$$ For node 2

$$\Rightarrow I_2 - v_2\left(\frac{1}{R_2}+\frac{1}{R_3}\right)+\frac{v_1}{R_2}=0$$

$$\Rightarrow I_2 - \frac{v_2-v_1}{R_2} - \frac{v_2}{R_3}=0$$

$$v_3\left(\frac{1}{R_1}+\frac{1}{R_4}\right)-\frac{v_1}{R_1}=-I_2$$ For node 3

$$\Rightarrow -v_3\left(\frac{1}{R_1}+\frac{1}{R_4}\right)+\frac{v_1}{R_1}-I_2=0$$

$$\Rightarrow -\frac{v_3-v_1}{R_1}-\frac{v_3}{R_4}-I_2=0$$

Finally, the three equations are:

$$-3v_1+2v_2+v_3=4$$
$$-4v_1+5v_2=12$$
$$-3v_1+5v_3=-18$$

In matrix from:

$$\begin{bmatrix}-3 & 2 & 1\\ -4 & 5 & 0\\ -3 & 0 & 5\end{bmatrix}\begin{bmatrix}v_1\\ v_2\\ v_3\end{bmatrix}=\begin{bmatrix}4\\ 12\\ -18\end{bmatrix}$$

$$\therefore \Delta = \begin{vmatrix}-3 & 2 & 1\\ -4 & 5 & 0\\ -3 & 0 & 5\end{vmatrix} = -3(25)-2(-20)+1(15)$$
$$= -75+40+15 = -100$$
$$\cancel{= -115+15}$$

$$\Delta_1 = \begin{vmatrix} 4 & 2 & 1 \\ 12 & 5 & 0 \\ -18 & 0 & 5 \end{vmatrix} = (4\times 25) - (2\times 60) + (1\times \cancel{15}\,90)$$

$$= 100 - 120 + \cancel{10}\,90 = 70.$$

$$\Delta_2 = \begin{vmatrix} -3 & 4 & 1 \\ -4 & 12 & 0 \\ -3 & -18 & 5 \end{vmatrix} = (-3\times 60) - (4\times -20) + 1(72+36)$$

$$= -180 + 80 + 108$$

$$\cancel{= 210 + 24 = -236}$$

$$= -180 + 188 = 8$$

$$\Delta_3 = \begin{vmatrix} -3 & 2 & 4 \\ -4 & 5 & 12 \\ -3 & 0 & -18 \end{vmatrix} = (-3\times -90) - 2(72+36) + (4\times 15) = 270 - 216 + 60 = 114$$

$$\cancel{= -45 / -48 \times 16} = 114$$

$$\cancel{= -93\ 466 =}$$

$$\therefore \cancel{v_1 = \frac{\Delta_1}{\Delta} = \frac{2}{-100} = +0.05 \text{ volts.}}$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{70}{-20} = -3.5\text{ V.}$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{8}{-20} = -0.4\text{ V.}$$

$$v_3 = \frac{\Delta_3}{\Delta} = \frac{114}{-20} = -5.7\text{ V.}$$

$$\therefore v = v_2 = -0.4\text{ V. (Ans).}$$

Notes:- The equation for any node can be written by direct inspection by using the following rule. For example, the 1st equation i.e. the eqn for node 1 of this problem is represented by.

1. Product of potential v_1 and $\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ i.e. sum of the ... resistances connected to this node

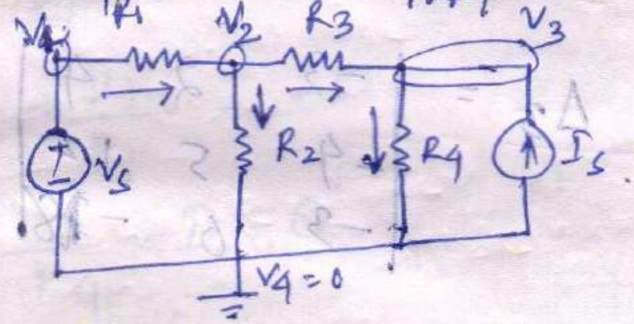
2. minus the ratio of adjoining potentials and the interconnecting resistances, i.e. $\left(-\frac{V_2}{R_2}\right) - \left(\frac{V_3}{R_1}\right)$.

3. All the above equated to the current supplied by the current source connected to this node. This current is taken positive if flowing into the node and negative if flowing out of it.

NODE ANALYSIS WITH VOLTAGE SOURCE :→

Let us consider the ckt. given below. Here V_1 is the dependent variables & V_2 & V_3 & V_4 are independent variables.

Here, $V_1 = V_S$.



Applying KCL at each nodes, we have:

$$\frac{V_1 - V_2}{R_1} - \frac{V_2}{R_2} - \frac{V_2 - V_3}{R_3} = 0$$

$$\Rightarrow \frac{V_S - V_2}{R_1} - \frac{V_2}{R_2} - \frac{V_2 - V_3}{R_3} = 0 \quad \text{(at node 1)}$$

At node 2: $\frac{V_2 - V_3}{R_3} + I_S - \frac{V_3}{R_4} = 0$

Here the nos. of equations required to solve the problem is $(n-m-1)$ where m is the nos. of nodes connected to a voltage source. So here $(4-1-1 = 2)$ 2 nos. of linear systems of equations as given above are required to solve the problems.

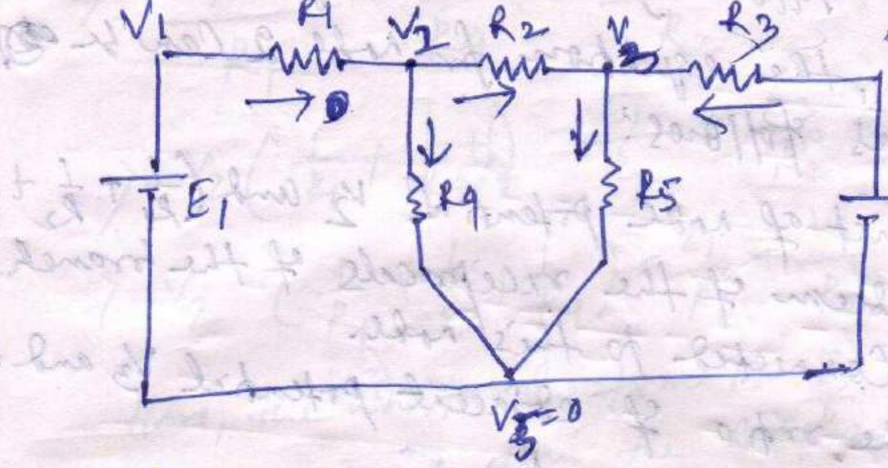
The above two equations can be written systematically as follows:

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) V_2 - \frac{1}{R_3} V_3 = \frac{1}{R_1} V_S$$

$$\& \quad -\left(\frac{1}{R_3}\right) V_2 + \left(\frac{1}{R_3} + \frac{1}{R_4}\right) V_3 = I_S$$

The resulting system of two equations in two unknowns can now be solved.

Let us consider another ckt. as given below.



Here, $V_1 = E_1$ & $V_4 = E_2$. V_1 & V_4 are dependent variables.

Nos. of equations required to solve is $(5-2-1) = 2$.

The two equations can be solved by applying KCL at node 2 and node 3.

At node 2.

$$\frac{V_1 - V_2}{R_1} - \frac{V_2}{R_4} - \frac{V_2 - V_3}{R_2} = 0$$

$$\Rightarrow \frac{E_1 - V_2}{R_1} - \frac{V_2}{R_4} - \frac{V_2 - V_3}{R_2} = 0$$

$$\Rightarrow V_2\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) - \frac{V_3}{R_2} = \frac{E_1}{R_1}$$

$$\Rightarrow V_2\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) - \frac{V_3}{R_2} - \frac{E_1}{R_1} = 0$$

At node 3

$$\frac{V_2 - V_3}{R_2} + \frac{V_4 - V_3}{R_3} - \frac{V_3}{R_5} = 0$$

$$\Rightarrow \frac{V_2 - V_3}{R_2} + \frac{E_2 - V_3}{R_3} - \frac{V_3}{R_5} = 0$$

$$\Rightarrow V_3\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) - \frac{V_2}{R_2} = \frac{E_2}{R_3}$$

$$\Rightarrow V_3\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) - \frac{V_2}{R_2} - \frac{E_2}{R_3} = 0$$

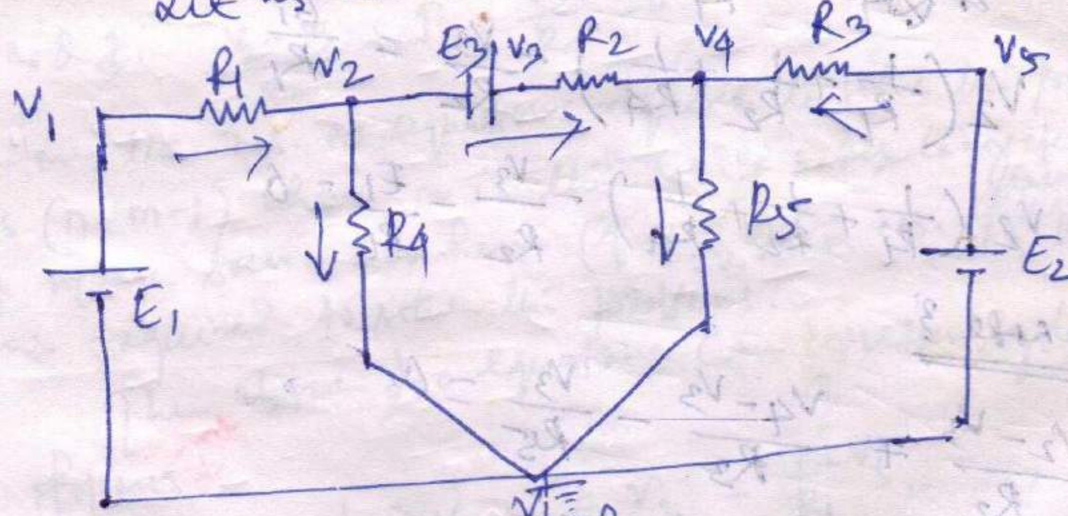
The above two equations can be written simply by inspection following the rules as given below. For instance, the equation for node 2 can be represented as follows:

1. The product of node potential V_2 and $\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}\right)$ i.e. the sum of the reciprocals of the branch resistance connected to this node.
2. Minus the ratio of adjacent potential V_3 and the interconnecting resistance R_2.
3. Minus ratio of adjacent battery voltage E_1 and interconnecting resistance R_1.
4. All the above set to zero.

~~If the batter~~

If viewed from the node if there is fall in potential, then the battery emf E_1 will be minus as in the above case and vice versa.

Let us consider another ckt. as given below.



Nos. of equations required to solve is $(6-3-1)=2$.

The two equations can be obtained by applying KCL at node 2 & node 4.

At node 2.

$$\frac{V_2-V_4}{\ldots}=0.$$

V_2

$$\Rightarrow \frac{E_1 - V_2}{R_1} - \frac{V_2}{R_4} - \frac{V_2 + E_3 - V_4}{R_2} = 0$$

$$\Rightarrow V_2\left(\frac{1}{R_1} + \frac{1}{R_4} + \frac{1}{R_2}\right) - \frac{E_1}{R_1} + \frac{E_3}{R_2} - \frac{V_4}{R_2} = 0$$

$$\Rightarrow V_2\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) - \frac{E_1}{R_1} - \frac{V_4}{R_2} + \frac{E_3}{R_2} = 0.$$

Here, the additional term is $\frac{E_3}{R_2}$ & it is positive because as viewed from the node 2, there is rise in potential & therefore E_3 is taken as positive.

At node 4:→

$$\frac{V_5 - V_4}{R_3} - \frac{V_4}{R_5} + \frac{V_2 + E_3 - V_4}{R_2} = 0.$$

$$\Rightarrow \frac{E_2 - V_4}{R_3} - \frac{V_4}{R_5} + \frac{V_2 + E_3 - V_4}{R_2} = 0$$

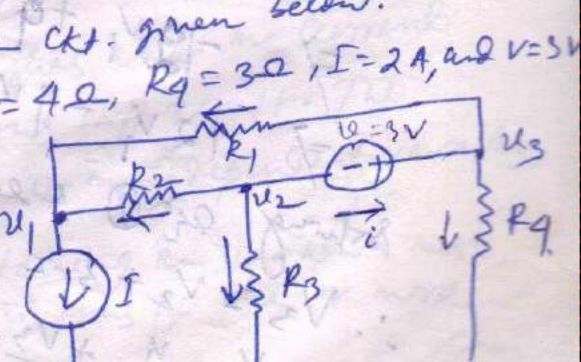
$$\Rightarrow V_4\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) - \frac{E_2}{R_3} - \frac{V_2}{R_2} - \frac{E_3}{R_2} = 0$$

Here, the additional term is $\frac{E_3}{R_2}$ & it is negative because as viewed from the node 4, there is fall in potential of the battery emf & therefore, E_3 is taken negative.

NODE ANALYSIS WITH BOTH VOLTAGE & CURRENT SOURCES:→

Ex → 3-6
R-91

Problem:→ Use node analysis to determine the current i flowing through the voltage source in the ckt. given below. Assume that $R_1 = 2\Omega$, $R_2 = 2\Omega$, $R_3 = 4\Omega$, $R_4 = 3\Omega$, $I = 2A$, and $V = 3V$



Ex:→3.8
R-95.

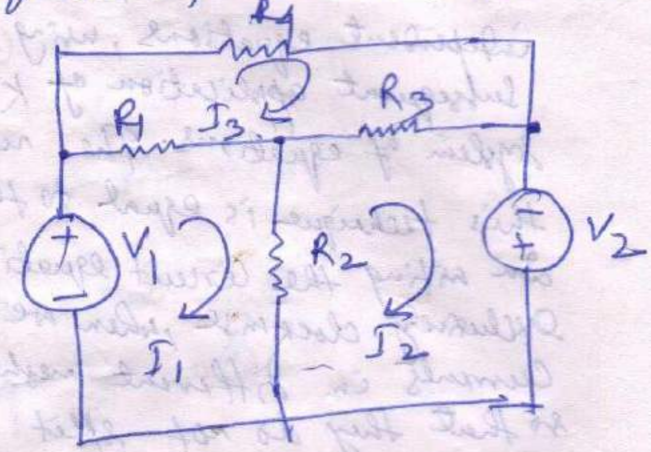
Problem :→

Write the mesh current equations for the circuit shown below.

Given that :

$V_1 = 12V$, $V_2 = 6V$, $R_1 = 3\Omega$,

$R_2 = 8\Omega$, $R_3 = 6\Omega$ & $R_4 = 4\Omega$.



Sol:→ Applying KVL to Mesh 1

$$V_1 - (I_1 - I_3)R_1 - (I_1 - I_2)R_2 = 0$$

$$\Rightarrow V_1 - I_1R_1 + I_3R_1 - I_1R_2 + I_2R_2 = 0$$

$$\Rightarrow (R_1 + R_2)I_1 - R_2I_2 - R_1I_3 = V_1 \quad -(1)$$

Applying KVL to Mesh 2

$$-(I_2 - I_3)R_3 + V_2 - (I_2 - I_1)R_2 = 0$$

$$\Rightarrow -I_2R_3 + I_3R_3 + V_2 - I_2R_2 + I_1R_2 = 0$$

$$\Rightarrow (R_2 + R_3)I_2 - R_2I_1 - R_3I_3 = V_2 \quad -(2)$$

$$\Rightarrow -R_2I_1 + (R_2 + R_3)I_2 - R_3I_3 = V_2$$

Applying KVL to Mesh 3

$$-I_3R_4 - (I_3 - I_2)R_3 - (I_3 - I_1)R_1 = 0$$

$$\Rightarrow -I_3R_4 - I_3R_3 + I_2R_3 - I_3R_1 + I_1R_1 = 0 \quad -(3)$$

$$\Rightarrow -R_1I_1 - R_3I_2 + (R_1 + R_3 + R_4)I_3 = 0$$

The three equations can be written in matrix form as follows.

$$\begin{bmatrix} (R_1 + R_2) & -R_2 & -R_1 \\ -R_2 & R_2 + R_3 & -R_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ 0 \end{bmatrix}$$

The generalized from of the above matrix can be written.

$$\begin{bmatrix} R_{11} & -R_{12} & -R_{13} \\ -R_{21} & R_{22} & -R_{23} \\ -R_{31} & -R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \Rightarrow [R][I] = [V]$$

$$\Rightarrow [I] = [R]^{-1}[V]$$

where $[R]^{-1} = \text{adj}\,R / \text{det}\,R$.

Where, R_{11}, R_{22} & R_{33} are the self resistances of the mesh 1, mesh 2 & mesh 3 respectively.

$R_{12} = R_{21}$ = Sum of all the resistances ~~connected~~ common to mesh ① & ②

$R_{13} = R_{31}$ = Sum of all the resistances common to mesh 1 and 3

$R_{23} = R_{32}$ = Sum of all the resistances common to mesh 2 and 3.

E_1, E_2 & E_3 are the sum of the emfs acting round the mesh of meshes 1, 2 & 3 respectively in the direction of current flow in the respective meshes.

Putting the values of resistances & emfs in eqn ①, ② & ③, we have

$$(3+8)I_1 - 8I_2 - 3I_3 = 12.$$

$$\Rightarrow 11I_1 - 8I_2 - 3I_3 = 12.$$

$$-8I_1 + 14I_2 - 6I_3 = 6.$$

$$-3I_1 - 6I_2 + 13I_3 = 0.$$

In matrix from, the above equations can be written as.

$$\begin{bmatrix} 11 & -8 & -3 \\ -8 & 14 & -6 \\ -3 & -6 & 13 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 6 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 11 & -8 & -3 \\ -8 & 14 & -6 \\ -3 & -6 & 13 \end{bmatrix}^{-1} \begin{bmatrix} 12 \\ 6 \\ 0 \end{bmatrix}$$

$$[R]^{-1} = \frac{\text{Adjoint of } R}{\text{det } R} = \frac{\text{Transpose of the Cofactor matrix}}{\text{Det } A}$$

$$\text{adj}[R] = \begin{bmatrix} (14\times13)-(6\times6) & -\{(-8\times13)-18\} & 48+42 \\ -(-104-18) & (11\times13)-9 & -\cancel{(48+42)}(-66-24) \\ (48+42) & -(-66-24) & (14\times11)-64 \end{bmatrix}^T$$

$$= \begin{bmatrix} 146 & \overset{122}{\cancel{86}} & 90 \\ 122 & 134 & -42 \\ 90 & 90 & 90 \end{bmatrix}^T = \begin{bmatrix} 146 & 122 & 90 \\ 122 & 134 & 90 \\ 90 & -42 & 90 \end{bmatrix}$$

$$|R| = 11(182-36)+8(-104-18)-3(48+42)$$

$$= 1606-976-270$$

$$= 360$$

$$\therefore [R]^{-1} = \frac{1}{360}\begin{bmatrix} 146 & 122 & 90 \\ 122 & 134 & 90 \\ 90 & -42 & 90 \end{bmatrix} = \begin{bmatrix} \frac{146}{360} & \frac{122}{360} & \frac{90}{360} \\ \frac{122}{360} & \frac{134}{360} & \frac{90}{360} \\ \frac{90}{360} & \frac{-42}{360} & \frac{90}{360} \end{bmatrix}$$

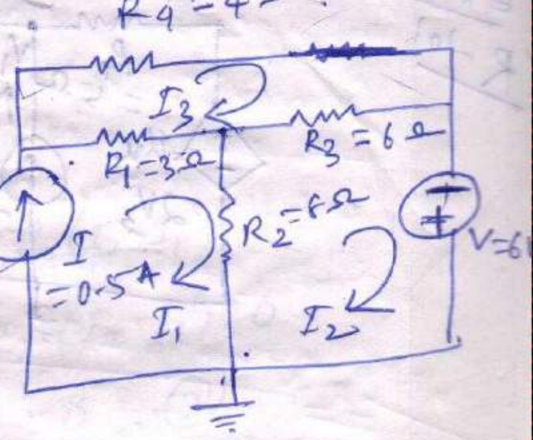
$$\therefore \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \frac{146}{360} & \frac{122}{360} & \frac{90}{360} \\ \frac{122}{360} & \frac{134}{360} & \frac{90}{360} \\ \frac{90}{360} & \frac{-42}{360} & \frac{90}{360} \end{bmatrix}_{3\times3} \begin{bmatrix} 12 \\ 6 \\ 8 \end{bmatrix}_{3\times1}$$

$$= \begin{bmatrix} 4.87+2.03 \\ 4.07+2.23 \\ 3-0.7 \end{bmatrix} = \begin{bmatrix} 6.9 \\ 6.3 \\ 2.3 \end{bmatrix} \quad \therefore I_1 = 6.9\,A,\; I_2 = 6.3\,A,\; I_3 = 2.3\,A$$

MESH ANALYSIS WITH VOLTAGE & CURRENT SOURCES :→

Ex:→3.10
R-99

Problem:→ Determine the branch currents and voltages across the resistances of the ckt given below using mesh current method.



Sol:→ $I_1 = I$.

Applying KVL to mesh 2 & 3, we have:

$\cancel{R_3 I_2 - V - R_2(I_2 - I_1) = 0}$

$\Rightarrow -R_3(I_2 - I_3) + V - R_2(I_2 - I_1) = 0$

$\Rightarrow V = R_2(I_2 - I) + R_3(I_2 - I_3)$

mesh 1

$\Rightarrow (R_2 + R_3)I_2 - R_2 I_1 - R_3 I_3 = V$.

and, $-R_4 I_3 - R_3(I_3 - I_2) - R_1(I_3 - I_1) = 0$

$\Rightarrow -R_4 I_3 - R_3(I_3 - I_2) - R_1(I_3 - I) = 0$

$\Rightarrow -R_1 I_1 - R_3 I_2 + (R_1 + R_3 + R_4) I_3 = 0$. mesh-2

Solving the above two equations:

$I_1 = I = 0.5\,A$, $I_2 = 0.95\,A$, $I_3 = 0.55\,A$.

Branch current through $R_1 = (I_3 - I_1) = 0.55 - 0.5 = 0.05\,A$.

" " " $R_2 = (I_2 - I_1) = 0.95 - 0.5 = 0.45\,A$.

" " " $R_3 = (I_2 - I_3) = 0.95 - 0.55 = 0.4\,A$.

" " " $R_4 = I_3 = 0.55\,A$.

The voltages across different resistances keeping in mind the reference node are found as follows:

$V_{R_1} = -(I_3 - I_1)R_1 = -0.05 \times 3 = -0.15\,V$

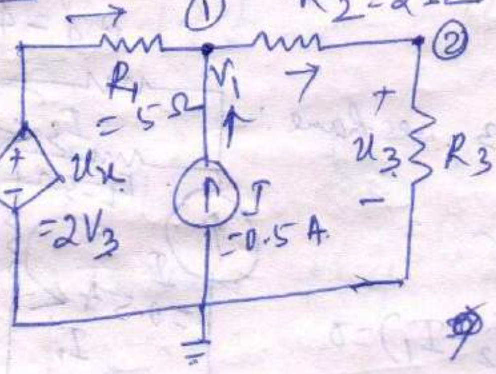
$\cancel{V_{R_1} = -(0.45 \times 8)}$

$= -0.45 \times 8 = -3.6\,V$.

$(I_2 - I_1)R_2 = -0.45 \times 8 = \ldots \quad V_{R_4} = I_3 R_4$

NODE AND MESH ANALYSES WITH CONTROLLED SOURCES:

Ex:- 3.12 R-101 Problem:- Determine the node voltages of the given ckt.



Applying KCL to node 1.

$$\frac{u_x - V_1}{R_1} + I - \frac{V_1}{R_2 + R_3} = 0$$

OR

$$\frac{u_x - V_1}{R_1} + I - \frac{V_1 - V_3}{R_2} = 0$$

$$\Rightarrow \frac{2V_3 - V_1}{R_1} + I - \frac{V_1 - V_3}{R_2} = 0 \Rightarrow V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_3\left(\frac{1}{R_2} + \frac{2}{R_1}\right) = I$$

$$\Rightarrow V_1\left(\frac{1}{5} + \frac{1}{2}\right) - V_3\left(\frac{1}{2} + \frac{2}{5}\right) = 0.5$$

$$\Rightarrow 7V_1 - 9V_3 = 5 \quad -①$$

Applying KCL to node 2.

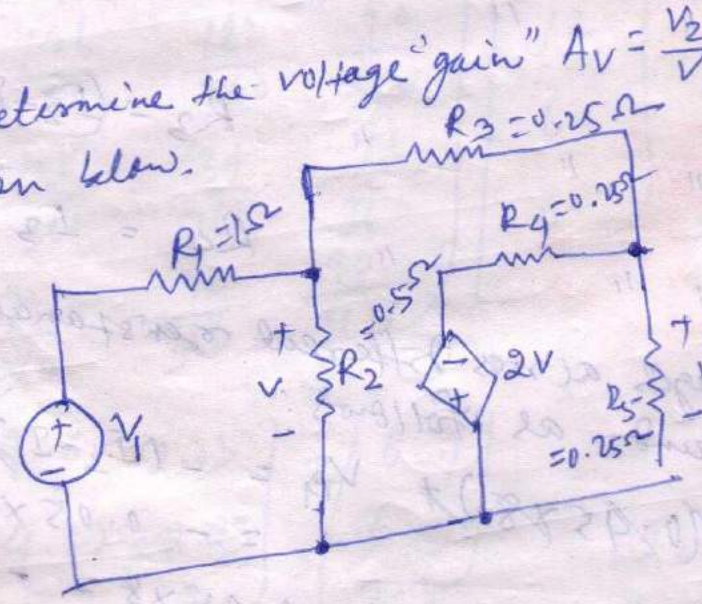
$$\frac{V_1 - V_3}{R_2} - \frac{V_3}{R_3} = 0.$$

$$\Rightarrow \frac{V_1}{R_2} - V_3\left(\frac{1}{R_2} + \frac{1}{R_3}\right) = 0. \Rightarrow \frac{V_1}{2} - V_3\left(\frac{1}{2} + \frac{1}{4}\right) = 0$$

$$\Rightarrow 2V_1 - 3V_3 = 0. \quad -②$$

Solving the above two equations we have: $V_1 = 5$ V & $V_3 = 3.33$ V.

Ex:- 3.13 R-102 Problem:- Determine the voltage "gain" $A_V = \frac{V_2}{V_1}$ in the circuit of fig. given below.



Applying KVL at each mesh, we have

<u>For mesh 1</u>:

$$V_1 - R_1 I_1 - R_2 (I_1 - I_2) = 0$$

$$\Rightarrow (R_1 + R_2) I_1 - R_2 I_2 = V_1 \quad \text{—(1)}$$

<u>For mesh 2</u>:

$$-R_2 I_1 + (R_2 + R_3 + R_4) I_2 - R_4 I_3 = 2V.$$

$$-R_2 I_1 + (R_2 + R_3 + R_4) I_2 - R_4 I_3 = -2 R_2 (I_2 - I_1)$$

$$\Rightarrow -R_2 I_1 + (R_2 + R_3 + R_4) I_2 - R_4 I_3 + 2R_2 (I_2 - I_1) = 0$$

$$\Rightarrow -R_2 I_1 + (R_2 + R_3 + R_4) I_2 - R_4 I_3 = 0 \quad \text{(2)}$$

$$\Rightarrow -3R_2 I_1 + (3R_2 + R_3 + R_4) I_2 - R_4 I_3 = 0$$

<u>For mesh 3</u>:

$$-R_4 I_2 + (R_4 + R_5) I_3 = -2V.$$

$$-R_4 I_2 + (R_4 + R_5) I_3 = -2 \times -R_2 (I_2 - I_1)$$

$$\Rightarrow -R_4 I_2 + (R_4 + R_5) I_3 = 0. \quad \text{(3)}$$

$$\Rightarrow 2R_2 I_1 - (2R_2 + R_4) I_2 + (R_4 + R_5) I_3 = 0.$$

Writing the above three equations in matrix form, we have:

$$\begin{bmatrix} R_1 + R_2 & -R_2 & 0 \\ -3R_2 & 3R_2 + R_3 + R_4 & -R_4 \\ 2R_2 & -(2R_2 + R_4) & R_4 + R_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix}$$

which can be written as $[R][I] = [V]$

with solution $[I] = [R]^{-1} [V]$

$$[R]^{-1} = \begin{bmatrix} 0.88 & 0.32 & 0.16 \\ 0.64 & 0.96 & 0.48 \\ 0.16 & 1.76 & 2.88 \end{bmatrix}$$

and therefore,

$$\therefore \begin{bmatrix} I_1 \\ I_2 \\ \ \end{bmatrix} = \begin{bmatrix} 0.88 & 0.32 & 0.16 \\ 0.64 & 0.96 & 0.48 \end{bmatrix} \begin{bmatrix} V_1 \\ 0 \\ 0 \end{bmatrix}$$

$I_1 = 0.88 V_1$

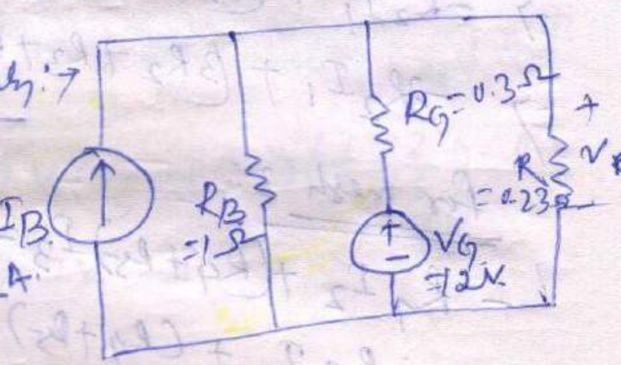
$I_2 = 0.64 V_1$

$I_3 = -0.16 V_1$

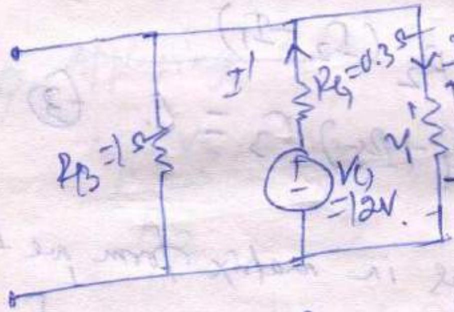
THE PRINCIPLE OF SUPERPOSITION:→

The principle of superposition states that in a linear circuit containing N sources, each branch voltage and current is the sum of N voltages and currents, each of which may be computed by setting ~~all~~ omitting all other sources except the source under consideration: omitted sources are replaced by ~~their~~ resistances equal to their internal resistances.

Ex:→3.15 Pb:→ Principle of Superposition:→
R-107. Determine the voltage R of the given circuit:



Soln:→ Considering the voltage source only:→



$$R_{eq} = \left(\frac{1 \times 0.23}{1+0.23} + 0.3\right) = 0.48699$$

$$\therefore I' = \frac{12}{0.48699} = 24.64116A$$

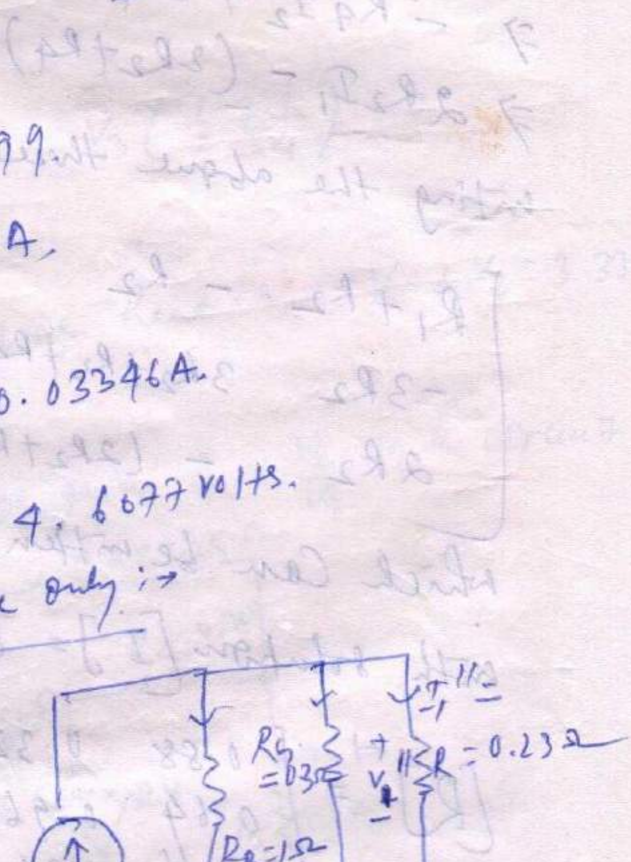
$$\& \ I_1' = \frac{24.64116 \times 1}{1.23} = 20.03346A$$

$$\therefore V_1' = 20.03346 \times 0.23 = 4.6077 \text{ volts.}$$

Considering the current source only:→

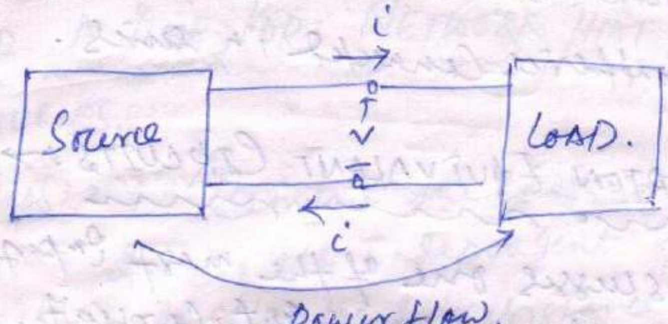
$$I_1'' = \frac{\frac{1}{R}}{\frac{1}{R_B} + \frac{1}{R_G} + \frac{1}{R}} I_B$$

$$= \frac{\frac{1}{0.23}}{\frac{1}{1} + \frac{1}{0.3} + \frac{1}{0.23}} \times 12$$

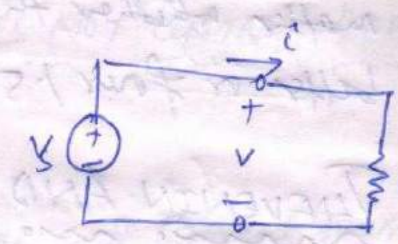


$= 1.34783 \times 12 = 1.5084 \times 12$ $\therefore V = V_1' + V_1''$

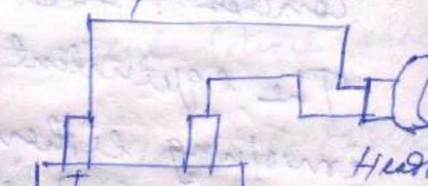
One-Port Networks and Equivalent Circuits:→



(a) Conceptual Representation

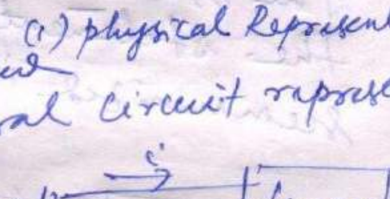


(b) Symbolic (circuit) representation



(c) physical Representation.

The above figures shows various forms for source-load representation. But, whatever the form chosen for source-load representation, each block - source or load - may be viewed as a two-terminal device, described by an i-v characteristics. This general circuit representation is shown in fig. given below.



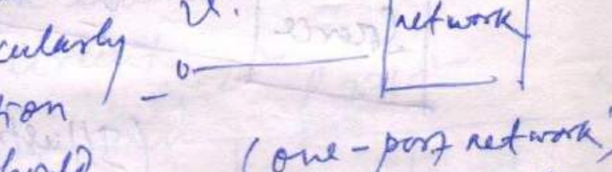
(one-port network).

This configuration is called a one-port network and is particularly useful for introducing the notion of equivalent circuits. It should be noted that the network given above is completely described by its i-v characteristic,

Equivalent Resistance Calculation:→

Ex:→3.16
R-109

Problem:→ Determine the source (load) current i in the circuit of given fig. using equivalent resistance

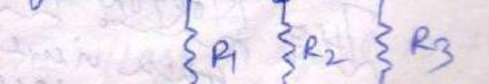


(Illustration of equivalent-circuit Concept).

Sol:→ Insofar as the source is concerned, the three parallel resistors appear identical to a single equivalent resistance of value

$$R = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}.$$

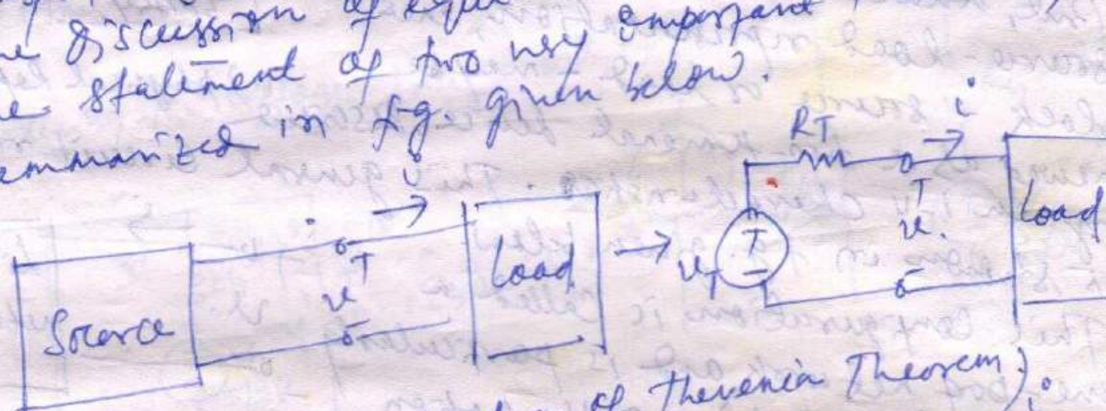
Thus we can replace the three load resistors with the single equivalent R as shown below, and calculate



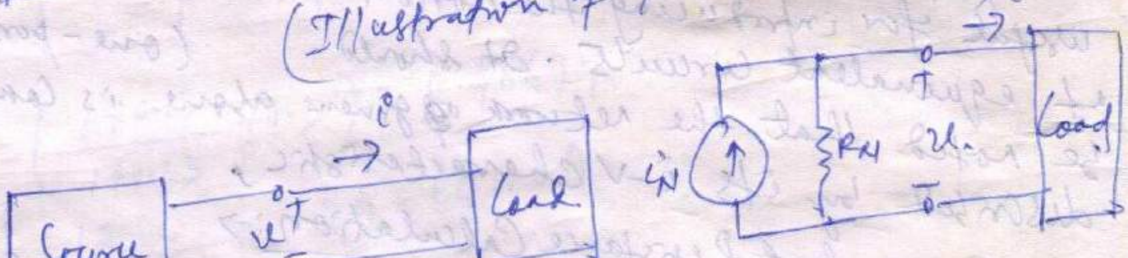
Similarly, insofar as the load is concerned, it would not matter whether the source consisted, say, of a single 6-V battery or four 1.5 V batteries connected in series.

THEVENIN AND NORTON EQUIVALENT CIRCUITS :→

This section discusses one of the most important concept : the concept of an equivalent circuit. The equivalent circuits fall into one of two classes, involving either voltage or current sources and (respectively) either series or parallel resistors. The discussion of equivalent circuits beginning with the statement of two very important theorems, summarized in fig. given below.



(Illustration of Thevenin Theorem)



(Illustration of Norton theorem)

THE THEVENIN THEOREM :→ When viewed from the load, any network composed of ideal voltage and current sources, and of linear resistors, may be represented by an equivalent circuit consisting of an ideal voltage source v_T in series with an equivalent resistance R_T.

THE NORTON THEOREM :→ When viewed from the load, any network composed of ideal voltage and current sources, and of linear

Determination of Norton or Thevenin Equivalent Resistance of a One-Port Network that does not contain Dependent Sources.

Steps:→ 1. Remove the load.

2. Zero all independent voltage and current sources.

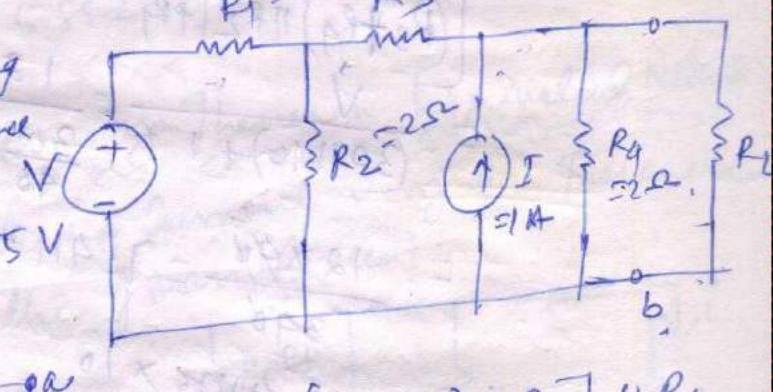
3. Compute the total resistance between load terminals, with the load removed. The resistance is equivalent to that which would be encountered by a current source connected to the circuit in place of the load.

This procedure yields a result that is independent of the load. This is a very desirable feature, since once the equivalent resistance has been identified for a source circuit, the equivalent circuit remains unchanged if we connect a different load.

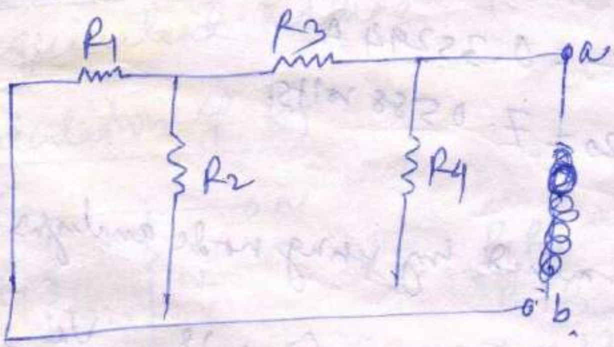
Thevenin Equivalent Resistance:→

Ex:→ 3/8 R-113.

Problem:→ Compute the Thevenin equivalent resistance seen by the load of the given circuit.



Sol:→ The circuit for determining the Thevenin Equivalent Resistance is obtained by removing the sources as given below.

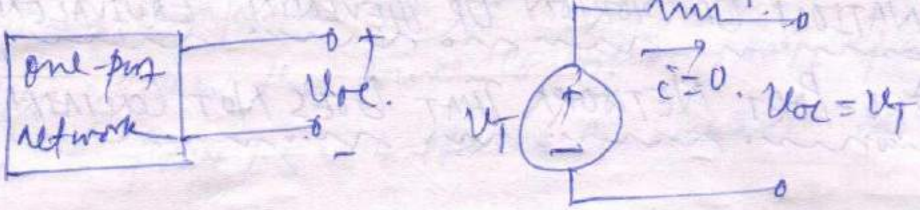


$$R_T = \left[(R_1 \parallel R_2) + R_3\right] \parallel R_4.$$
$$= \left[(2 \parallel 2) + 1\right] \parallel 2$$
$$= 1\,\Omega.$$

The Thevenin and Norton equivalent resistances are one and the same quantity: $\boxed{R_T = R_N}$

Computing the Thevenin Voltage:→

The equivalent (Thevenin) source voltage is equal to the



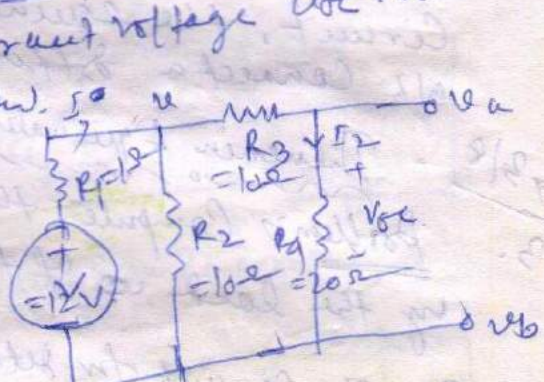
(Equivalence of open-circuit and Thevenin voltage).

Steps :→
1. Remove the load, leaving the load terminals open-circuited.
2. Define the open-circuit voltage V_{oc} across the open load terminals.
3. Apply any preferred method (i.e. node analysis) to solve for V_{oc}.
4. The thevenin voltage is $V_T = V_{oc}$.

Thevenin Equivalent voltage (open-circuit voltage) :→

EX:-3.19
R-115

Problem :→ Compute the open circuit voltage V_{oc} in the circuit of fig. given below.



$$I = \frac{V}{[(R_3+R_4) \parallel R_2] + R_1}$$

$$= \frac{V}{(30 \parallel 10) + 1} = \frac{12}{\frac{300}{40} + 1}$$

$$= \frac{12 \times 40}{340} = 1.41176\,A$$

$$\therefore I_2 = \frac{1.41176 \times 10}{40} = 0.35294\,A$$

$$\therefore V_{oc} = 0.35294 \times 20 = 7.0588 \text{ volts.}$$

Alternate method

V_{oc} can be determined by using node analysis method as given below:

$$\frac{v}{R_1+R_2+R_3} - \frac{V_a}{R_3} - \frac{V}{R_1} = 0 \quad -(1) \Rightarrow \frac{v}{21} - \frac{V_a}{10} - \frac{12}{1} = 0$$

$$\& \quad \frac{V_a}{R_3+R_4} - \frac{v}{R_3} = 0 \Rightarrow \frac{V_a}{30} - \frac{v}{10} = 0 \quad -(2)$$

$$\Rightarrow v = \frac{V_a}{30} \times 10 = \frac{V_a}{3}$$

$$v\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)-\frac{V_a}{R_3}-\frac{V}{R_1}=0.$$

$$\Rightarrow v\left(1+\frac{1}{10}+\frac{1}{10}\right)-\frac{V_a}{10}-12=0$$

$$\Rightarrow v\left(\frac{12}{10}\right)-\frac{V_a}{10}=12$$

$$\Rightarrow \frac{12v-V_a}{10}=12$$

$$\Rightarrow 12v-V_a=120 \quad \text{—(1)}$$

$$V_a\left(\frac{1}{R_3}+\frac{1}{R_4}\right)-\frac{v}{R_3}=0$$

$$\Rightarrow V_a\left(\frac{1}{10}+\frac{1}{20}\right)-\frac{v}{10}=0$$

$$\Rightarrow \frac{3V_a}{20}=\frac{v}{10}$$

$$\Rightarrow v=\frac{3}{2}V_a \quad \text{—(2)}$$

Now eqn (1) becomes $12\times\frac{3}{2}V_a-V_a=120$

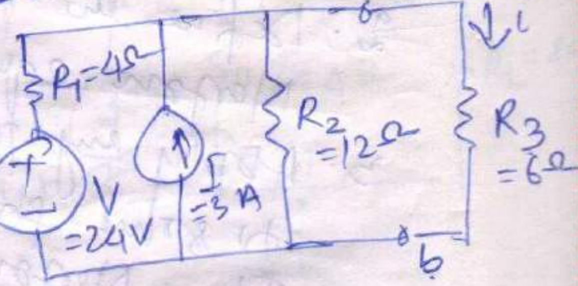
$\Rightarrow 18V_a-V_a=120 \Rightarrow V_a=7.05882$ volts.

$\therefore V_{OC}=V_a-V_b=7.05882$ volts.

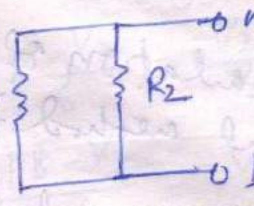
Load current calculation by Thevenin Equivalent Method:

Ex:-> 3.20 R-117

Problem:-> Compute the load current i by the Thevenin equivalent method of the given circuit.

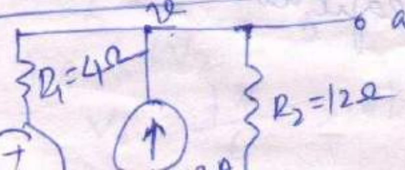


Soln:-> Calculation of R_T



$$R_T=\frac{R_1R_2}{R_1+R_2}=\frac{4\times12}{4+12}=3\Omega$$

Calculation of V_T.

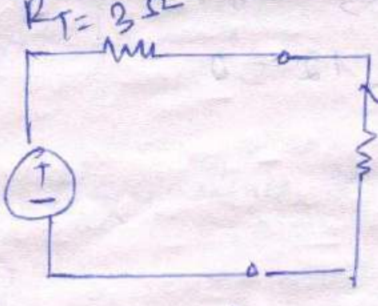


$$\cancel{\frac{V_a}{R_1}-\frac{V}{R_1}}\ V_a\left(\frac{1}{R_1}+\frac{1}{R_2}\right)-\frac{V}{R_1}=3$$

$$\Rightarrow V_a\left(\frac{1}{4}+\frac{1}{12}\right)-\frac{24}{4}=3$$

= 27 volts

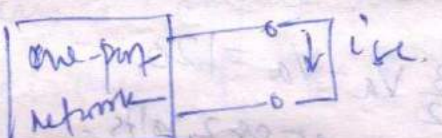
Thevenin's Equivalent Ckt

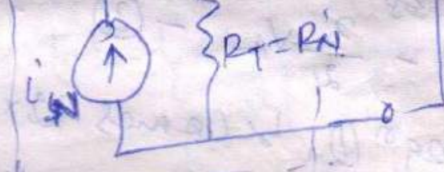


$$\therefore i = \frac{V_T}{R_T + R_3} = \frac{27}{3+6} = 3\text{ A.}$$

Computing the Norton Current :→

The Computation of the Norton equivalent current is very similar in concept to that of the Thevenin voltage. The Norton equivalent current is equal to the short-circuit current that would flow if the load were replaced by a short-circuit.





(Illustration of Norton equivalent Circuit)

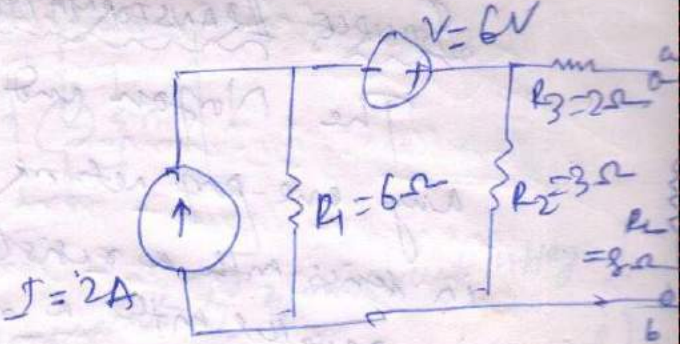
Steps :→

1. Replace the load with a short-circuit.
2. Define the short-circuit current i_{sc} to be the Norton equivalent current.
3. Apply any preferred method (i.e. node analysis) to solve for i_{sc}.
4. The Norton current is $i_N = i_{sc}$.

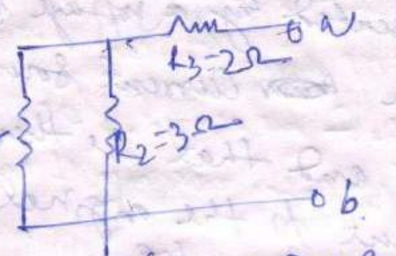
Norton Equivalent Ckt & Calculation of load current by Norton equivalent method :→

Ex:-3.21
R-119

Problem :→ Determine the load current of the given ckt by Norton equivalent method.



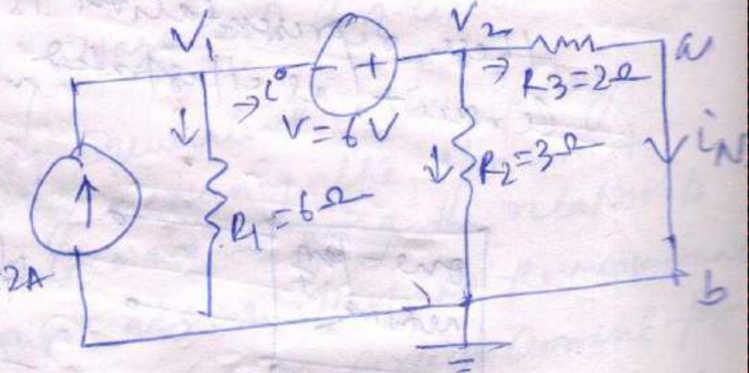
Calculation of $R_N = R_T$



$$R_N = R_T = (R_1 \parallel R_2) + R_3$$

$$= \frac{6 \times 3}{6+3} + 2 = 4\,\Omega$$

Calculation of I_N.



V_1

$$I - \frac{V_1}{R_1} - i = 0$$

$$\Rightarrow 2 - \frac{V_1}{6} - i = 0 \quad \text{--- (1)}$$

$$i - \frac{V_2}{R_2} - \frac{V_2}{R_3} = 0 \quad \text{--- (2)}$$

But, $V_2 = V_1 + V = V_1 + 6$

$$\Rightarrow V_1 = V_2 - V$$

$\therefore$ Eqn (1) becomes $2 - \frac{V_2 - V}{6} - i = 0$

$$\Rightarrow \frac{V_2 - V}{6} = 2 - i$$

$$\Rightarrow V_2 = 12 - 6i + V$$

$$\Rightarrow 6i = 12 + V - V_2$$

$$\Rightarrow i = \frac{12 + 6 - V_2}{6} = \frac{18 - V_2}{6}$$

Substituting this in eqn (2)

$$\frac{18 - V_2}{6} - \frac{V_2}{R_2} - \frac{V_3}{R_3} = 0$$

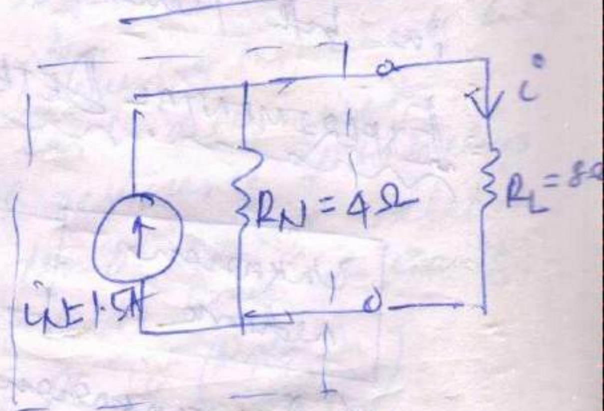
$$\Rightarrow \frac{18 - V_2}{6} - V_2\left(\frac{1}{3} + \frac{1}{2}\right) = 0$$

$$\Rightarrow \frac{18 - V_2}{6} - \frac{5}{6}V_2 = 0$$

$$\Rightarrow 18 - 6V_2 = 0$$

$$V_2 = \frac{18}{\;} = 3\text{ volts}$$

Norton Equivalent circuit and Calculation of Load current i



$$\therefore\; i = \frac{1.5 \times 4}{12}$$

$$= \frac{6}{12} = 0.5\text{ A.}$$

SOURCE TRANSFORMATIONS :-

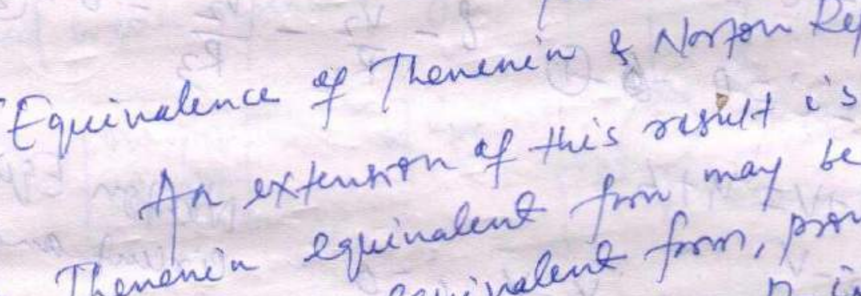
The Norton and Thevenin theorems state that any one-port network can be represented by a voltage source in series with a resistance, or by a current source in parallel with a resistance, and that either of these representations is equivalent to the original circuit, as illustrated in the following fig.

Thevenin Equivalent   Norton Equivalent.

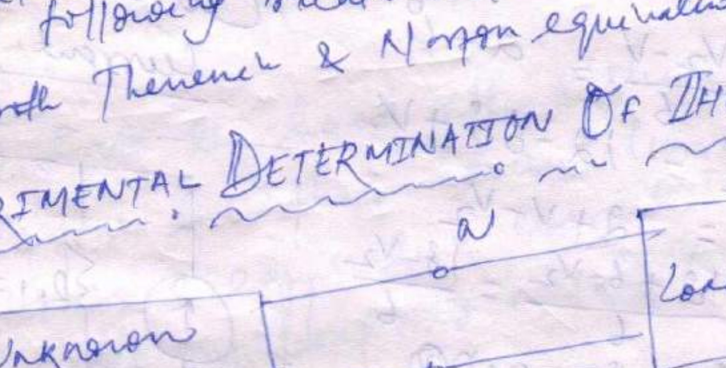
(Equivalence of Thevenin & Norton Representations)

An extension of this result is that any circuit in Thevenin equivalent form may be replaced by a circuit in Norton equivalent form, provided that we use the following relationship: $v_T = R_T i_N$ = open circuit voltage in both Thevenin & Norton equivalent circuit.

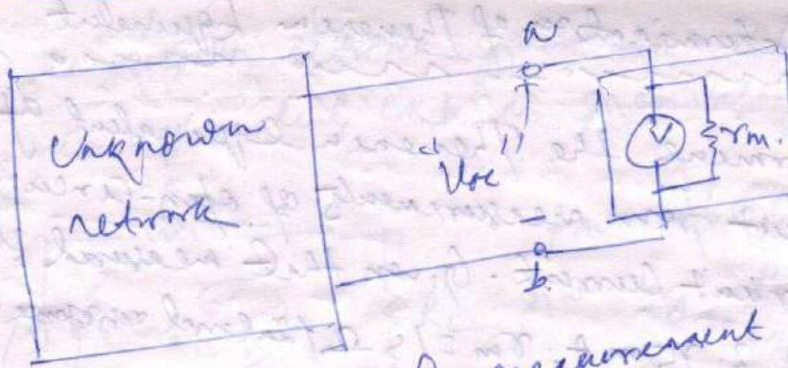
EXPERIMENTAL DETERMINATION OF THEVENIN & NORTON EQUIVALENTS



(An unknown network connected to a load)



Network Connected for measurement of short-circuit



(Network Connected for measurement of open-circuit voltage.)

(Measurement of open-circuit voltage and short-circuit current)

(Measurement of open-circuit Equivalent Circuit of any.

The Thevenin & Norton Equivalent Circuit of any network may be determined by measuring the open circuit voltage & short-circuit current and using the relationship $R_T = \frac{v_T}{i_N}$. The fig. given above illustrates the measurement of the open-circuit voltage and short-circuit current for any arbitrary network connected to any load. Because of the nonideal nature of any practical measuring instrument (i.e. here in this case both the ammeter and voltmeter has some internal resistance), the short-circuit current & the true open circuit voltage is given by the following expressions:

$$i_N = \text{"}i_{sc}\text{"}\left(1+\frac{r_m}{R_T}\right)$$

$$v_T = \text{"}v_{oc}\text{"}\left(1+\frac{R_T}{r_m}\right)$$

This relationship is true because for an practical ideal ammeter $r_m = 0$ & for an ideal voltmeter $r_m = \infty$.

If the internal resistance of the measuring instruments are known, then i_N & v_T can be easily determined. However, in practice, the internal resistance of voltmeters is sufficiently high to be considered infinite relative to the equivalent resistance of most practical circuits.

Therefore $v_T = \text{"}v_{oc}\text{"} = R_T i_N = R_T \text{"}i_{sc}\text{"}\left(\frac{R_T + r_m}{R_T}\right)$

$\Rightarrow v_T = v_{oc} = i_{sc}(R_T + r_m)$

$\Rightarrow \boxed{\frac{v_{oc}}{i_{sc}} - r_m = R_T}$

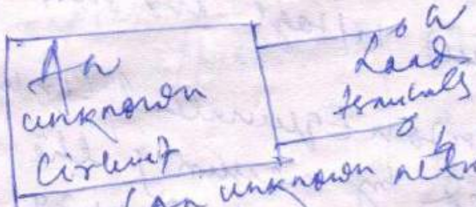
$\boxed{v_T = v_{oc}}$

$(i_{sc}\, r_m)$

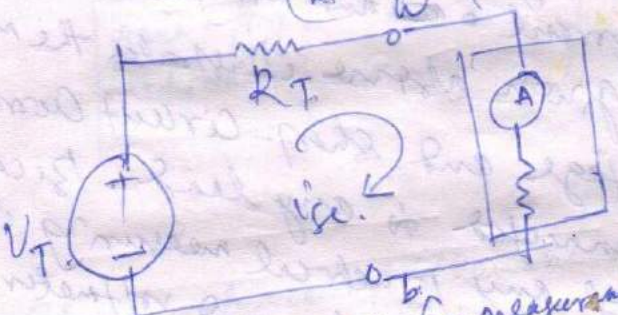
Experimental Determination of Thevenin Equivalent Circuit :-

Problem :- Determine the Thevenin equivalent of an unknown circuit from measurements of open-circuit voltage and short-circuit current. Given that measured $V_{oc} = 6.5V$, measured $i_{sc} = 3.75\ mA$, $r_m = 15\ \Omega$ (Internal resistance of ammeter)

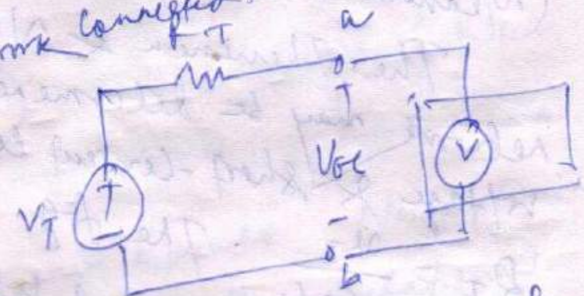
Sol :-



(An unknown network connected to a load)



Network connected for measurement of short-circuit current (practical ammeter).



Network connected for measurement of open-circuit voltage (ideal voltmeter)

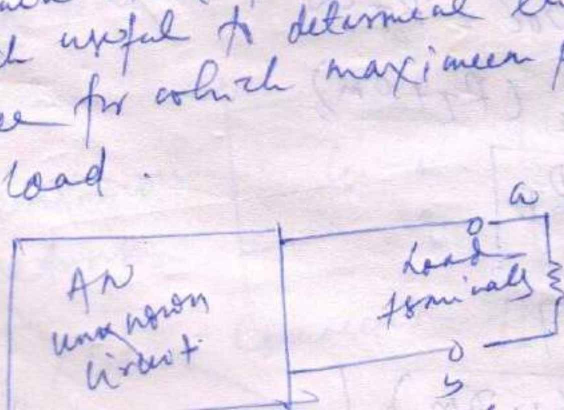
$V_{oc} = V_T = 6.5\ V$.

$R_T = \frac{V_{oc}}{i_{sc}} - r_m = \frac{6.5}{(3.75\times10^{-3})} - 15 = 1{,}718\ \Omega$.

$i_{A1} = i_{sc}\left(1+\frac{r_m}{R_T}\right) = (3.75\times10^{-3})\left(1+\frac{15}{1{,}718}\right) A$.

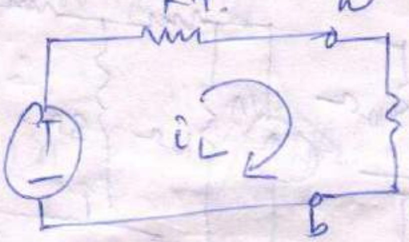
MAXIMUM POWER TRANSFER :-

Any linear resistive circuit can be reduced to its Thevenin or Norton Equivalent and this concept is very much useful to determine the value of the load resistance for which maximum power can be transferred to the load.



(... to a load)

The complicated network given above can be represented by its Thevenin equivalent ckt as given below.



Power absorbed by the load R_L is given by $P_L = i_L^2 R_L$

& $i_L = \frac{V_T}{R_L + R_T}$.

$$\therefore P_L = \left(\frac{V_T}{R_L+R_T}\right)^2 R_L = \frac{V_T^2}{(R_L+R_T)^2} R_L$$

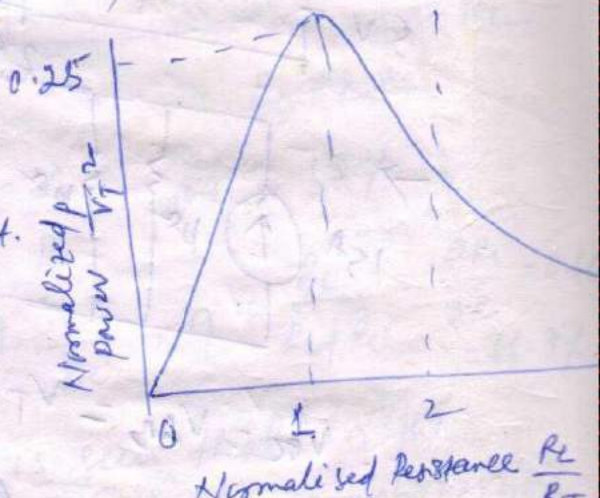
The value of R_L that maximizes the expression for P_L (assuming that V_T and R_T are fixed), can be obtained ~~as follows~~ by solving.

$$\frac{dP_L}{dR_L} = 0 \Rightarrow \frac{d}{dR_L}\left[\frac{V_T^2}{(R_L+R_T)^2} R_L\right] = 0$$

$$\Rightarrow V_T^2 (R_L+R_T)^2 - 2(R_L+R_T) V_T^2 R_L = 0$$

$$\Rightarrow R_L^2 + R_T^2 + 2R_L R_T - 2R_L^2 - 2R_L R_T = 0.$$

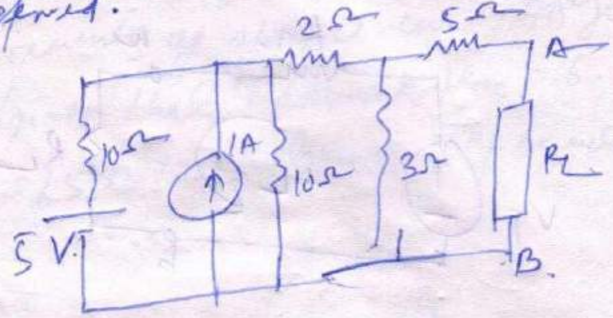
$$\Rightarrow \boxed{R_L = R_T}$$



If the circuit is simply a practical ~~source~~ voltage source with int. resistance r_i connected to a load of resistance R_L as given below, then the maximum power transfer to the load will take place when ~~source~~ load resistance equals the ~~load~~ source resistance, i.e. for maximum power transfer the equivalent source and load resistances must be matched.

for maximum power transfer $\boxed{R_L = r_i}$.

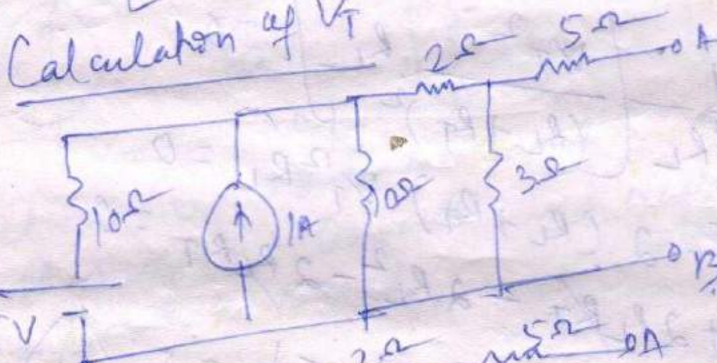
Ex:- 2.116 BLT-163 In the circuit given below obtain the condition for maximum power transfer to the load R_L. Hence determine the maximum power transferred.

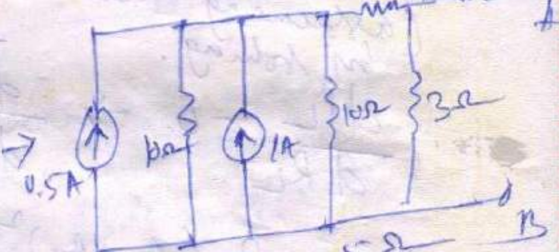


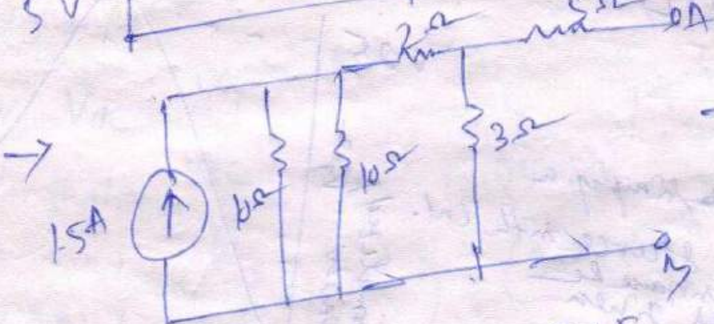
Soln:- Calculation of R_{Th} :-

$$R_T = \left[\left[(10 \parallel 10) + 2\right] \parallel 3\right] + 5 = \frac{21}{10} + 5 = 7.1\,\Omega$$

Calculation of V_T

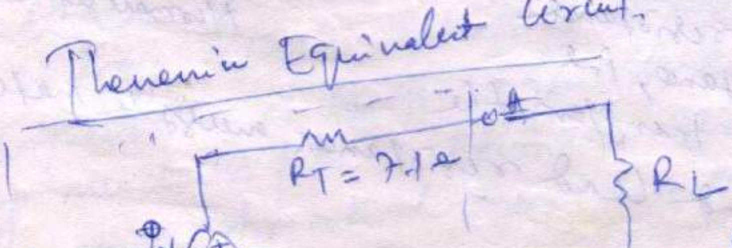






$$\therefore V_{OC} = V_{AB} = V_T = \frac{1.5 \times 5}{10} \times 3 = 2.25\,\text{volts}$$

Thevenin Equivalent Circuit.



$\therefore$ Maximum power transfer

$R_L = R_T = 7.1\,\Omega$.

maximum power transferred

$$= \left(\frac{V_T}{R_L + R_T}\right)^2 R_L$$

$$= \frac{V_T^2}{4R_L^2} \times R_L$$

$$= \frac{(2.25)^2}{4 \times 7.1} = 0.178$$

POWER TRANSFER EFFICIENCY :→

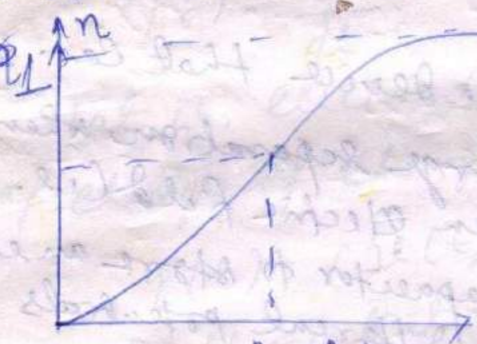
Power transfer efficiency $\eta = \dfrac{\text{Power supplied to the load}}{\text{Total power supplied by the voltage source}}$

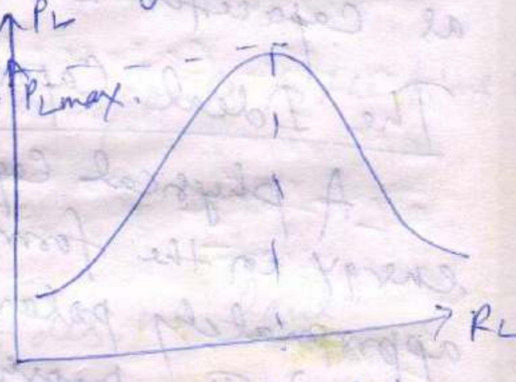
$$\Rightarrow \eta = \frac{P_L}{P_S} = \frac{I^2 R_L}{I^2(R_L + R_i)} = \frac{R_L}{R_L + R_i}$$

$\eta = 1$, when $R_L = \infty$

$= 0.5$, when $R_L = R_i$.

It means that under maximum ~~transf~~ power transfer conditions, the power transfer efficiency is only 50%.





When $R_L = R_i$, $P_L = \frac{1}{4}\frac{V_s^2}{R_i} = 0.25\frac{V_s^2}{R_i}$, $\eta = 50\%$.

When $R_L = 2R_i$, $P_L = \frac{2}{9}\frac{V_s^2}{R_i} = 0.22\frac{V_s^2}{R_i}$, $\eta = \frac{R_L}{R_L + R_i} = \frac{2R_i}{3R_i} = 0.66 = 66.7\%$

~~from above it is s~~

From the above calculation it is seen that in the 2nd case power is only 3% less than its maximum possible value, whereas the power transfer efficiency has improved from ~~0.5~~ 50% to 66.7% i.e. by 16.7%.

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