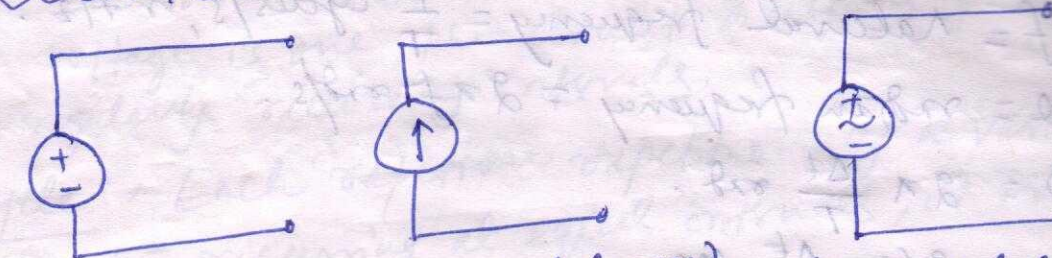
AC FUNDAMENTALS

TIME-DEPENDENT SIGNAL SOURCES :→



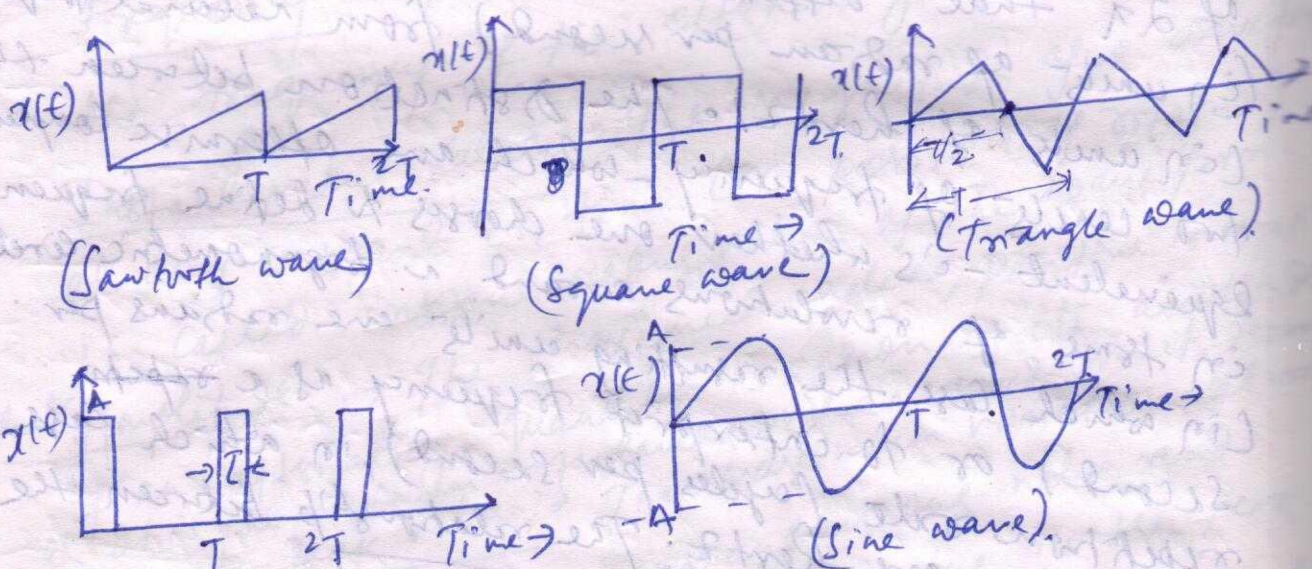
(Generalized time-dependent sources) (Sinusoidal source)

Fig. Time-dependent signal sources

One of the most important classes of time-dependent signals is that of periodic signals. A periodic signal is a signal that satisfies the equation

$$x(t) = x(t + nT), \quad n = 1, 2, 3, \ldots$$

where T is the period of $x(t)$.

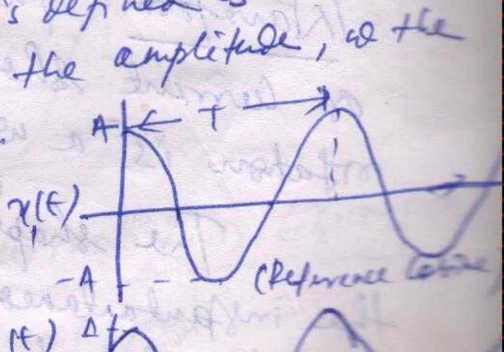


(Sawtooth wave) (Square wave) (Triangle wave)

(Pulse Train) (Sine wave)

Fig. Periodic signal waveforms

Sinusoidal waveforms constitute by far the most important class of time-dependent signals. The fig. given below depicts the relevant parameters of a sinusoidal waveform. A generalized sinusoid is defined as $x(t) = A\cos(\omega t + \phi)$ where A is the amplitude, ω the radian frequency, and ϕ the phase.



$x_1(t) = A\cos\omega t$

$x_2(t) = A\cos(\omega t - \phi)$

where,

f = natural frequency = $\frac{1}{T}$ cycles/s, or Hz.

ω = radian frequency = $2\pi f$ rad/s.

$\phi = 2\pi \frac{\Delta t}{T}$ rad.

$= 360 \frac{\Delta t}{T}$ deg.

The phase shift ϕ permits the representation of an arbitrary sinusoidal signal. Thus, a sine wave in terms of cosine wave can be represented simply by introducing a phase shift of $\pi/2$ rad:

$A \sin \omega t = A \cos(\omega t - \pi/2)$

It is important to be aware of the factor of 2π that differentiates radian frequency (in units of radian per second) from natural frequency (in units of hertz). The distinction between the two units of frequency — which are otherwise completely equivalent — is whether one chooses to define frequency in terms of revolutions around a trigonometric circle (in which case the resulting units are radians per second) or to interpret frequency as a ~~repetiti~~ repetition rate (cycles per second), in which case the units are hertz. The relationship between the two is $\boxed{\omega = 2\pi f \text{ radian frequency}}$.

WAVEFORM TERMS AND DEFINITIONS: →

Waveform: The variation of a quantity such as voltage or current shown on graph to a base of time or rotation is a waveform.

OR

The shape of the curve obtained by plotting the instantaneous values of voltage or current as the ... time as abscissa is called its

Alternating current or Voltage :→ An alternating current or voltage is one the circuit direction of which reverses at regularly recurring intervals.

Cycle :→ Each ~~repetion~~ repetition of a variable quantity, recurring at equal intervals, it termed a cycle.

OR

One complete set of positive and negative values of alternating quantity is known as cycle.

Period :→ The time taken by an alternating quantity to complete one cycle is called its period (T).

Frequency :→ The number of cycles/second is called the frequency of the alternating quantity.

Instantaneous Value :→ The magnitude of a waveform at any instant in time (or position of rotation) is called instantaneous value of the wave form.

Peak value or **Amplitude** :→ The maximum value, positive or negative of an alternating quantity is known as its amplitude.

OR It is the maximum instantaneous value measured from its zero value.

Peak-to-peak value :→ The maximum variation between the maximum positive instantaneous value and the maximum negative instantaneous value is the peak-to-peak value. For a sinusoidal wave form, the peak-to-peak value is twice the peak value. The peak-to-peak value is denoted as E_{pp} or V_{pp} or I_{pp}.

* Generation of Alternating Voltage & Current AND EQUATION

In next page.

PHASE:→ By phase of an alternating current is meant the fraction of the time period of that alternating current which has elapsed since the current last passed through the zero position of reference.

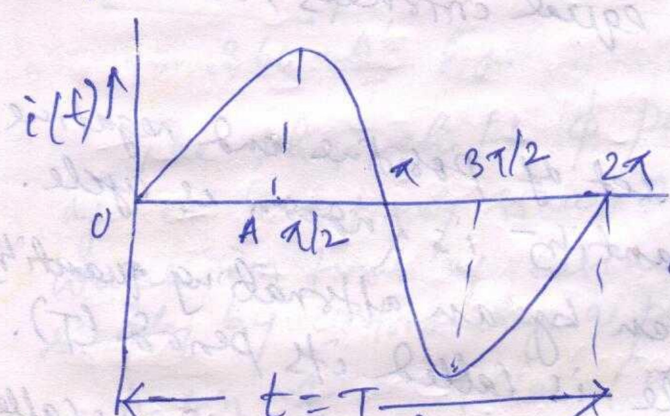


Fig (a)

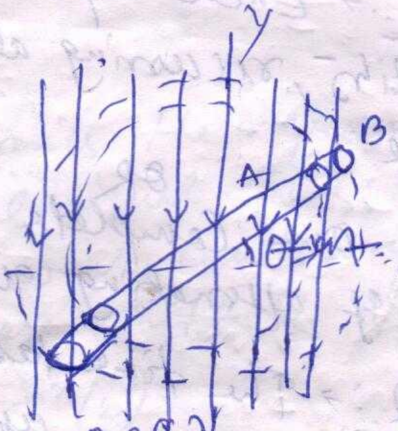


Fig (b)

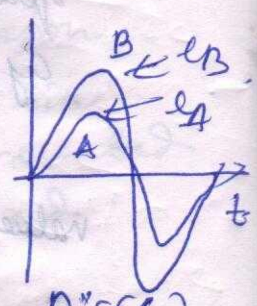


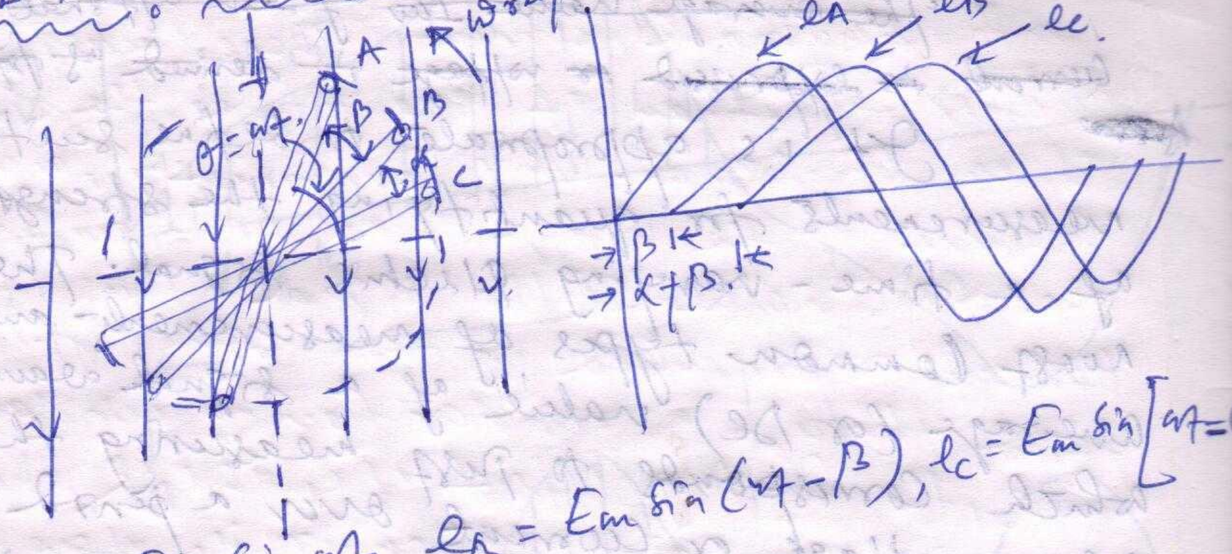
Fig (c)

For example, the phase of current at point A is T/4 second or in terms of angle, it is $\pi/2$ radians. Similarly, the phase of the alternating emf induced in coil A and B at the instant shown in Fig. (b) is ωt, which is, therefore, called its phase angle. In Fig. (c) e_A and e_B are in phase with each other as both e_A and e_B are reaching their zero and maximum values at the same time.

Two or more alternating quantities are said to be in phase if ~~both the~~ all the alternating quantities pass through their zero and maximum values at the same time.

In electrical engineering, we are more concerned with relative phases or phase differences between different alternating quantities, rather than with their absolute phase.

PHASE DIFFERENCE :→

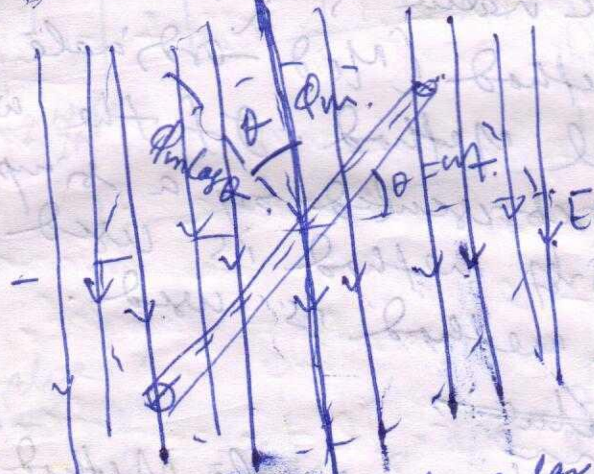


$e_A = E_m \sin \omega t$, $e_B = E_m \sin(\omega t - \beta)$, $e_C = E_m \sin[\omega t - ($

+ sign → Leading

− sign → Lagging.

* Generation of Sinusoidal Alternating Voltage & Current AND Eq



Let us consider a rectangular coil, having N turns and rotating in a uniform magnetic field, with an angular velocity of ω radian/second, as shown in the fi… Let time be measured from the X-axis. At any … the flux linkages of the coil are $N\Phi = N\Phi_m \cos \omega t$. Hence, according to Faraday's laws of Electromagnetic induction, the value of the emf induced at this in… (i.e when $\theta = \omega t$) or the instantaneous value of the induced emf is $e = -\frac{d}{dt}(N\Phi)$ volts

$= -N \frac{d}{dt}(\Phi_m \cos \omega t)$ volts $= \omega N \Phi_m \sin \omega t$ volt $= \omega N \Phi_m$ …

when $\theta = 90^\circ$, $e = \omega N \Phi_m = E_m$.

$e = E_m \sin \theta = E_m \sin \omega t$ volts

Average Value & RMS Value:→

~~The average value I_{av} of an alternating current is expressed as voltage is derived as follows.~~

It is appropriate to define suitable measurements for quantifying the strength of a time-varying electric signal. The most common types of measurements are the average (or DC) value of a signal waveform which corresponds to just measuring the mean voltage or current over a period of time - and the root-mean-square (or rms) value which takes into account the fluctuation of the signal about its average value.

There are two methods to determine the average value & RMS value, i.e. 1) Graphical Method (Mid-ordinate Method) and 2) Analytical Method. If the waveform can not be represented by a simple mathematical expression first method is used, otherwise the second method is used.

~~Average Value:~~→

1) Graphical (Mid-ordinate Method):→

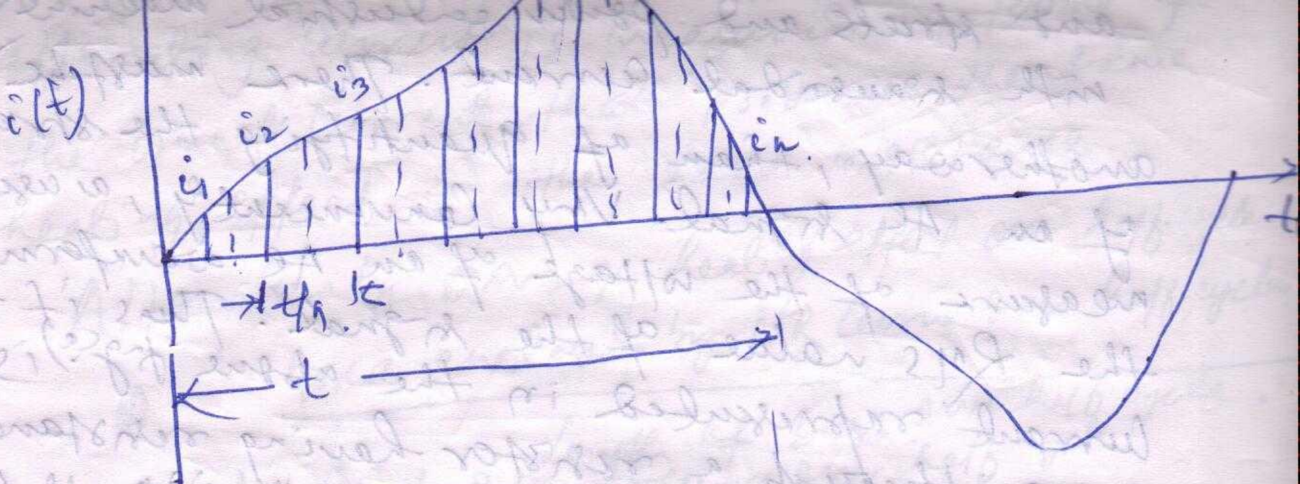
Average value:→

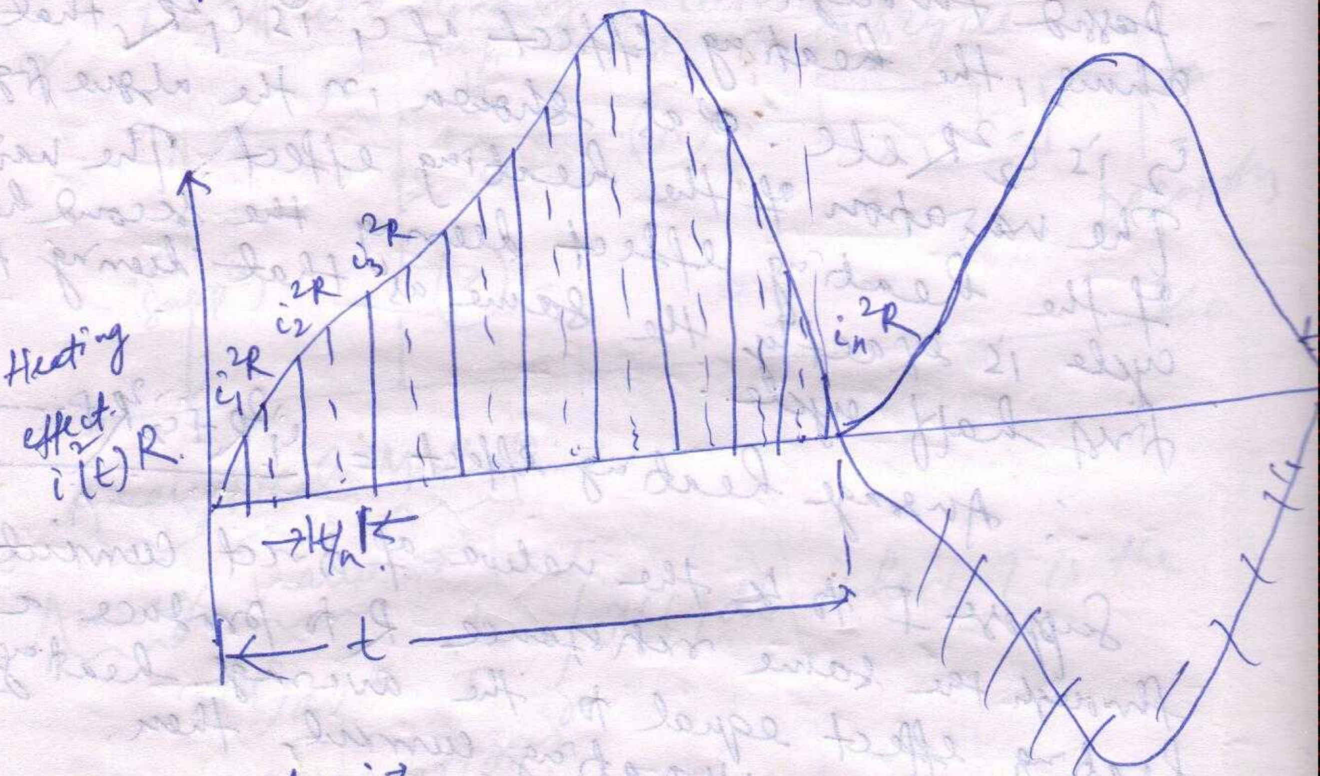
$$I_{av} = \frac{i_1 + i_2 + \cdots + i_n}{n}$$

or, Alternatively,

$$I_{av} = \frac{\text{Area enclosed over half-cycle}}{\text{Length of base over half-cycle}}$$

Average value of a symmetrical waveform over ... is zero. However, the average ... over half-cycle is not zero.





RMS value:-

$$I^2R = \frac{i_1^2R + i_2^2R + \cdots + i_n^2R}{n}$$

$$\Rightarrow I = \sqrt{\frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}}$$

In A.C. work, however, the average value is of comparatively little importance because the average value of a sinusoidal alternating voltage or current is zero over a complete cycle is zero. However, the average power of a sinusoidal alternating voltage or current is not zero. Otherwise, it would be impossible to illuminate households and streets and power industrial machinery with sinusoidal current. There must be another way, than of quantifying the strength of an AC signal. Very conveniently, a useful measure of the voltage of an AC waveform is the RMS value of the signal. Thus if the current represented in the above fig(a) is passed through a resistor having resistance R ohms, the heating effect of i_1 is i_1^2R, that of i_2 is i_2^2R etc. as shown in the above fig. (b). The variation of the heating effect. The variation of the heating effect during the second half of the cycle is exactly the same as that during the first half cycle.

$\therefore$ Average heating effect $= \dfrac{i_1^2R + i_2^2R + \cdots + i_n^2R}{n}$

Suppose I to be the value of direct current through the same resistance R to produce a heating effect equal to the average heating effect of the alternating current, then.

$$I^2R = \frac{i_1^2R + i_2^2R + \cdots + i_n^2R}{n}$$

$$\therefore I = \sqrt{\frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}}$$

= square root of the mean of the square of the current.

= root-mean-square (rms) value of the current.

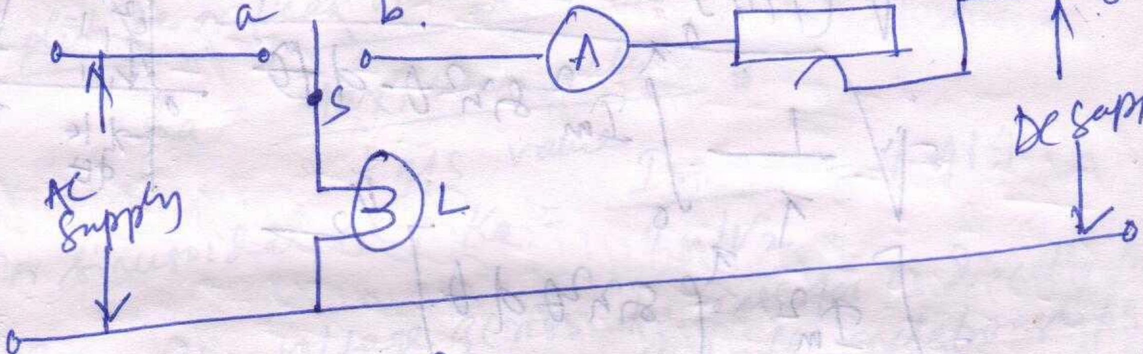
This quantity is also termed the effective of the current.

Hence, the RMS or Effective value of an alternating current is measured in terms of the direct current that produces the same heating effect in the same resistance.

Alternatively Average heating effect over half-cycle i.e.

$$= \frac{\text{Area enclosed by } i^2R \text{ curve over half cycle}}{\text{Length of base over half cycle.}}$$

or $$I = \frac{\text{Area enclosed by } i^2 \text{ curve over half cycle.}}{\text{Length of base over half cycle.}}$$

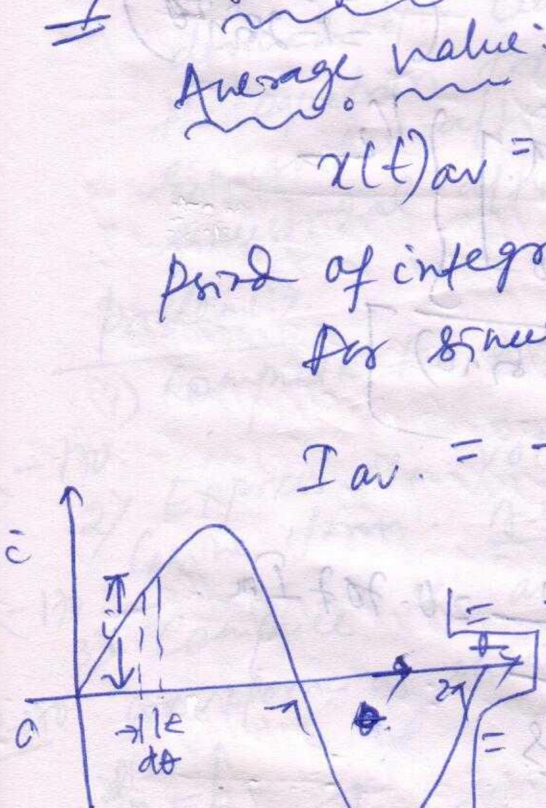


2) Analytical Method:→

Average value:→ $x(t)_{av} = \frac{1}{T}\int_0^T x(t)\,dt$ where T is the period of integration.

For sinusoidal waveform,

$$I_{av} = \frac{1}{(T/2)}\int_0^{(T/2)} i(t)\,dt$$ (The average value over a half-cycle)



$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\,d\theta$$

$$= -\frac{I_m}{\pi}\cos\theta\Big|_0^{\pi} = \frac{-I_m}{\pi}(\cos\pi - \cos 0)$$

$$= \frac{2I_m}{\pi} = 0.637 I_m.$$

RMS value :-

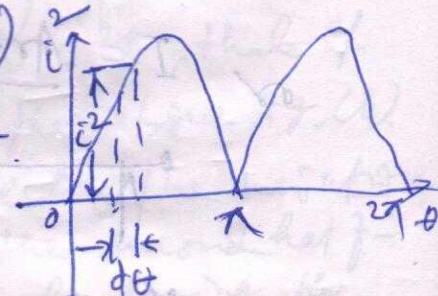
$$x_{RMS} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt}$$

where T is the period of integration.

For sinusoidal waveform,

$$I_{eff} \text{ or } I_{RMS} = \sqrt{\frac{1}{(T/2)}\int_0^{(T/2)} i^2(t)\,dt}$$

$$= \sqrt{\frac{1}{(T/2)}\int_0^{(T/2)} I_m^2 \sin^2\omega t \; d(t)}$$



$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta \; d\theta}$$

$$= \left[\frac{I_m^2}{\pi}\int_0^{\pi} \sin^2\theta \, d\theta\right]^{1/2}$$

$$= \left[\frac{I_m^2}{\pi}\times\frac{1}{2}\int_0^{\pi}(1-\cos 2\theta)\,d\theta\right]^{1/2} \quad \left(\because \cos 2\theta = 1 - 2\sin^2\theta\right)$$

$$= \left\{\frac{I_m^2}{2\pi}\left[d\theta\Big|_0^{\pi} - \frac{1}{2}\sin 2\theta\Big|_0^{\pi}\right]\right\}^{1/2}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\left[\pi - \frac{1}{2}(\sin 2\pi - \sin 0)\right]}$$

$$= \left[\frac{I_m^2}{2\pi}\times\pi\right]^{1/2} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m .$$

Notation to represent various values:

Instantaneous values :→ e, v, i

Maximum values :→ E_m, V_m, I_{max}

E, V, I.

Notes :→ The RMS value is always greater than the average value except for a rectangular wave, in which case the heating effect remains constant so that the average and the RMS values are the same.

Form factor :→

$$K_f = \frac{\text{RMS value}}{\text{Average value}}$$

For sinusoidal A.C., $K_f = \frac{I_m/\sqrt{2}}{2I_m/\pi} = 1.11$.

Amplitude or Peak or Crest factor :→

$$K_a = \frac{\text{Maximum value}}{\text{RMS value}}$$

For sinusoidal AC, $K_a = \frac{I_m}{I_m/\sqrt{2}} = 1.414$.

Notes :→ Since the voltage across the resistor is directly proportional to the current, it follows that the relationships derived for currents also apply to voltages. Hence, the equation for Average value, RMS value for the sinusoidal alternating current also holds good for sinusoidal alternating voltage.

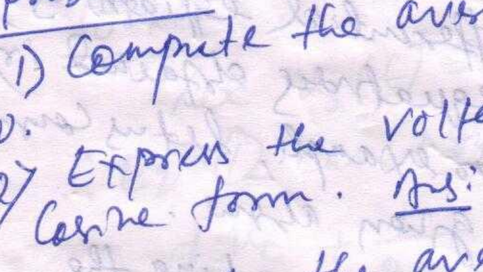
Problems :→

1) Compute the average value of the signal $x(t) = 10\cos(100t)$ Ans $= 0$

R-170. 2) Express the voltage $v(t) = 155.6\sin(377t + \pi/6)$ in cosine form. Ans :→ $155.6\cos(377t - \pi/3)$.

R-170. 3) Compute the average & RMS value of the sawtooth waveform as shown in the fig. given below.

R-170 Ans :→ $V_{av} = 2.5$ volts, $V = 2.89$ volts.



$V_{av} = \frac{\frac{1}{2} \times 10 \times 5}{10} = 2.5$ volts

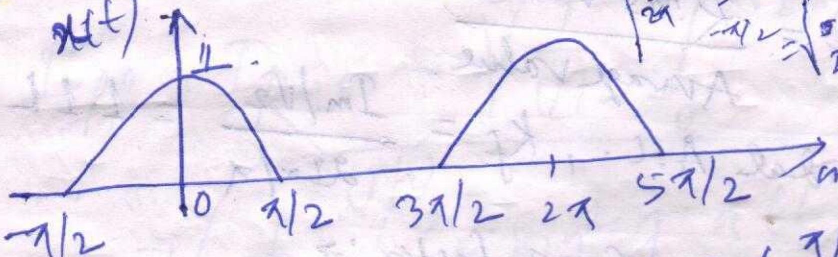
$V_{RMS} = \sqrt{\frac{1}{10}\int_0^{10}\left(\frac{1}{2}t\right)^2 dt} = \sqrt{\frac{1}{120} * t^3\Big|_0^{10}} = \sqrt{\frac{1}{120} \times 1000} = 2.89$

4) Compute the average value of the shifted triangle wave shown below. v(t) in volts. Ans :→ $V_{av} = 1.5$ volts.

5) Compute the RMS value of the sinusoidal current
R-172 $i(t) = I_m \cos t$.

Ans: $I = \frac{I_m}{\sqrt{2}}$.

6) Find the RMS value of the half cosine wave
R-172 shown in the fig. given below.



$$\sqrt{\frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} \cos^2 t \, dt}$$

$$= \sqrt{\frac{1}{2\pi}\cdot\frac{1}{2}\int_{-\pi/2}^{\pi/2}(1+\cos 2t)\,dt}$$

$$= \sqrt{\frac{1}{4\pi}\left[t + \frac{\sin 2t}{2}\right]_{-\pi/2}^{\pi/2}}$$

$$= \sqrt{\frac{1}{4\pi}\left[\pi + \frac{1}{2}(0-0)\right]}$$

$$= \sqrt{\frac{1}{4}} = \frac{1}{2} = 0.5\,V.$$

$\cos 2t = 1 - 2\sin^2 t$
$= 1 - 2(1-\cos^2 t)$
$= 1 - 2 + 2\cos^2 t$
$= 2\cos^2 t - 1$
$\therefore \cos^2 t = \frac{1+\cos 2t}{2}$

$x(t) = \cos t$ for $-\pi/2 < \omega t < \pi/2$ $\quad \omega = 1$

$= 0$ for $\pi/2 < \omega t < 3\pi/2$

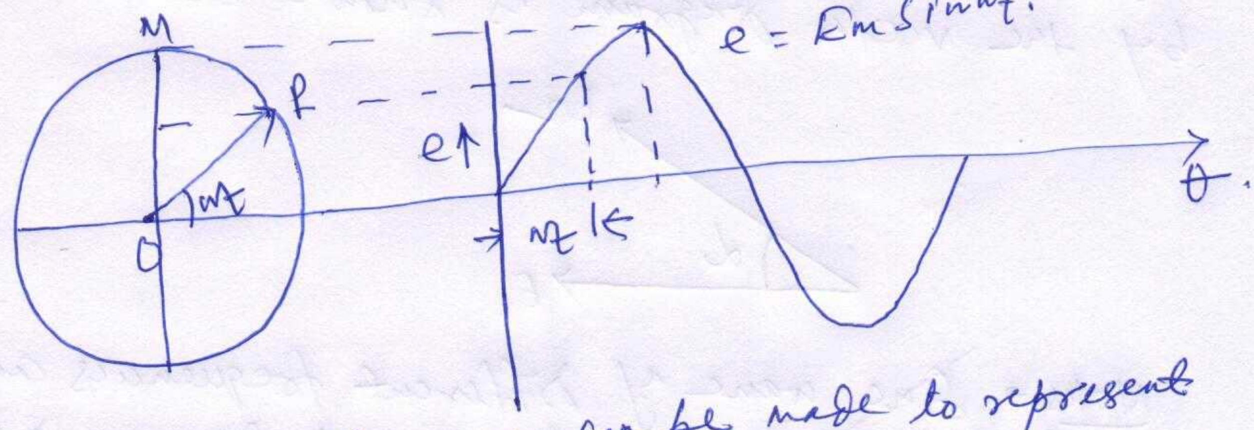
Representation of Alternating Quantities by vectors:→

AC computations are based on the assumption of sinusoidal voltages and currents. But it is difficult to continuously handle the instantaneous values in the form of equations like $e = E_m \sin \omega t$.

Hence, a conventional method is to employ vector methods for representation of sine waves. These vectors may be manipulated instead of the sine functions. Vectors are a short hand for the representation of alternating voltages and currents and their use greatly simplifies the solutions of computations in a.c. work.

The alternating ~~current~~ voltage is represented by the equation $e = E_m \sin \omega t$. As shown in the following fig., the projection of op and Y-axis at any instant gives the instantaneous value of the alternating voltage.

$$OM = op \sin \omega t \Rightarrow e = E_m \sin \omega t.$$



Hence, a line op can be made to represent an alternating voltage or current if it satisfies the following conditions:

i) Its length should be equal to the maximum value of sinusoidal alternating voltage/current to a suitable

ii) It should be in the horizontal position at the same instant as the alternating voltage/current is zero and increasing.

iii) Its angular velocity should be such that it completes one revolution in the same time as taken by the alternating quantity to complete one cycle.

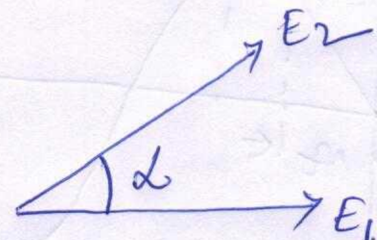
However, it is very common practice to draw vector diagrams using R.M.S. values of alternating voltages/currents instead of using maximum values. But, in that case, the projection of the rotating vector on the Y-axis does not give the instantaneous values of the alternating voltage/current.

Let us consider two alternating emfs represented by the equations:

$e_1 = E_m \sin \omega t$

and $e_2 = E_m \sin(\omega t + \alpha)$.

The two alternating emfs can be represented by the vector diagram as shown below.



Notes:→ Sine wave of different frequencies cannot be represented on the same vector diagram in a still picture because due to difference in speed of different vectors, the phase angles between them will be continuously changing.

Mathematical Representation of vectors :→

Various forms of representing vector quantities are described below.

i) Rectangular form

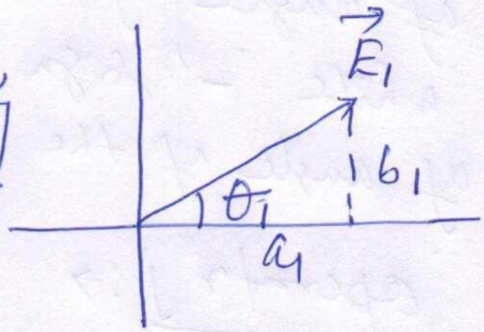
ii) Trigonometrical form.

iii) Exponential form.

iv) Polar form.

i) Rectangular form :→

$\boxed{\vec{E_1} = a_1 + j b_1.}$ In general, $\boxed{\vec{E} = a + jb}$



Numerical value of vector E_1 is $\sqrt{a_1^2 + b_1^2}$ & its angle with the x-axis is given by $\theta_1 = \tan^{-1} \frac{b_1}{a_1}$.

ii) Trigonometrical form :→

In the above fig.

$a_1 = E_1 \cos\theta_1$ & $b_1 = E_1 \sin\theta_1$

$\therefore \vec{E} = E_1 (\cos\theta_1 + j \sin\theta_1)$

In general, $\boxed{\vec{E} = E (\cos\theta + j \sin\theta)}$

iii) Exponential form :→

$e^{\pm j\theta} = \cos\theta \pm j \sin\theta.$

$\boxed{\vec{E} = E e^{\pm j\theta}.}$

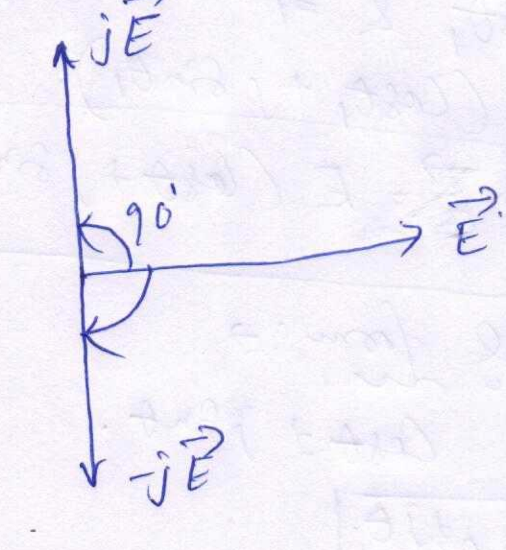
iv) Polar form: →

$$\boxed{\vec{E} = E\angle \pm\theta}$$

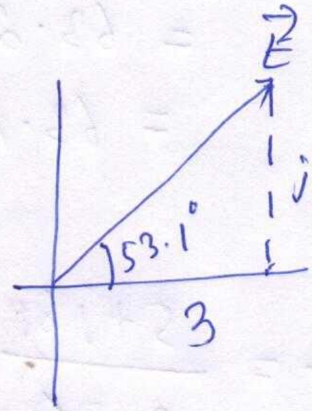
- where E is the magnitude of the vector & θ is the angle that the vector is making with the reference axis or X-axis. '+' sign indicates anticlockwise measurement of angle of the vector from the reference axis while '–' sign indicates the clockwise measurement of angle of the vector from the reference axis.

Operator j: →

When operator 'j' is operated on any vector $\vec{E}$; it rotates the vector $\vec{E}$ in anticlockwise direction by an angle 90°. Similarly when '–j' is operated on any vector $\vec{E}$, it rotates the vector $\vec{E}$ in clockwise direction by an angle 90° as shown below.



Problems:→ write the equivalent exponential and polar forms of vector $3+j4$ & illustrate the same by means of vector diagram.



Sol^n:→ Let the vector $\vec{E} = 3+j4$

$\therefore$ Magnitude of $\vec{E}$, $E = \sqrt{3^2+4^2}$
$= 5$

Angle of $\vec{E}$ with the reference axis i.e. X-axis,

$\theta = \tan^{-1}\frac{4}{3} = 53.1°$

$\therefore$ Exponential form, $\vec{E} = 5e^{j53.1°}$

& polar form, $\vec{E} = 5\angle 53.1°$

Notes:→ For addition & subtraction of vectors rectangular form is used whereas for multiplication & division of vectors polar form is used.

Problems:→ Perform the addition, subtraction, multiplication & division of the two vectors given by: $\vec{E_1} = 5+j8$ & $\vec{E_2} = 3-j6$.

Sol^n:→ $\vec{E_1}+\vec{E_2} = (5+j8)+(3-j6) = 8+j2 = 8.24\angle 14°$

$\vec{E_1}-\vec{E_2} = (5+j8)-(3-j6) = 2+j14 = 14.14\angle 81.87°$

(Real parts are added/subtracted & Imaginary parts are added/subtracted).

$\cancel{\frac{\vec{E_1}}{\vec{E_2}} = \frac{5+j8}{3-j6}}$

$$\vec{E_1} \cdot \vec{E_2} = (5+j8)(3-j6)$$
$$= (9.43\angle 58^\circ)(6.7\angle -63.43)$$
$$= 63.2\angle -5.43$$ (Magnitudes are multiplied and angles are added).
$$= 63.13 - j6.$$

$$\frac{\vec{E_1}}{\vec{E_2}} = \frac{5+j8}{3-j6} = \frac{9.43\angle 58^\circ}{6.7\angle -63.43^\circ} = 1.4\angle 121.43^\circ$$

(Magnitudes are divided and angles of the denominator is subtracted from the angle of numerator).

$$\Rightarrow \frac{\vec{E_1}}{\vec{E_2}} = 1.4\angle 121.43 = -0.73 + j1.2.$$

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